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Mathematical Modeling as A Catalyst for Equitable Mathematics Instruction: Preparing Teachers and Young Learners with 21st Century Skills

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Abstract: This case study focuses on a team of teachers and students in a Lesson Study, focused on using mathematical modeling (MM) to make significant decisions to design and plan for a sustainable edible garden in their community. We examined (a) how teachers develop students' capacity to engage in mathematical modeling, while attending to equitable teaching practices; and (b) how teachers' view of teaching through mathematics modeling changed after unit implementation. We found that teachers were deliberate in employing specific structures, routines, and tools to attend to equitable participation, when eliciting student thinking in the modeling process. We found that teachers' view of mathematics modeling changed as they recognized how MM allowed for (a) integration of important mathematics concepts while giving students ownership of the mathematics; (b) opportunity to assess both content and 21st century process skills; and (c) positive energy that came from both students and teachers when teaching through the use of mathematical modeling. A promising strategy for preparing our youth for rigorous mathematics and skills to solve ill-structured problems is by integrating mathematical modeling in early elementary grades to develop critical 21st century skills and a productive disposition towards problem posing and problem solving.

Keywords: mathematical modeling; equitable instruction; 21st century skills; elementary mathematics instruction



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1. Introduction

For this Special Issue on “Futuring Mathematics Education,” we focused on sharing innovative teaching practices that show promise in preparing our early learners from diverse backgrounds with critical 21st-century skills. The youngest of the school-age students in our study will be adults in 2035 and will live in a world that demands problem solving and critical thinking skills more than ever before. Our complex world requires global citizens to have creativity, critical thinking, communication, and collaboration skills, and to be ready and willing to tackle novel and challenging problems. The emerging future requires mathematics educators to look beyond the immediate horizon and broaden the realm of school mathematics to “recognize the role that mathematics plays in the world” [1].

The Organization for Economic Co-Operation and Development (OECD) [2] that administers the Program for International Student Assessment (PISA) to fifteen-year-olds recently published a new Framework for PISA 2021. The updated PISA framework responds to the mathematics necessary for a rapidly changing world, driven by new technologies “in which citizens are creative and engaged, making non-routine judgments for themselves and the society in which they live.” According to PISA’s definition [2] (p. 7), mathematical literacy is an individual’s capacity to reason mathematically and to

formulate, employ, and interpret mathematics to solve problems in a variety of real-world contexts. It includes concepts, procedures, facts, and tools to describe, explain, and predict phenomena. It helps individuals know the role that mathematics plays in the world and make well-founded judgments and decisions that are needed by constructive, engaged, and reflective 21st century citizens.

PISA's focused definition suggests that mathematics education needs to focus on higher-order activities, such as those involved in Mathematical Modeling (e.g., understanding real-world situations, transferring them into mathematical models, and interpreting mathematical results) and Computational Thinking (e.g., understanding to decompose problems, recognizing patterns, thinking algorithmically, and abstracting from computation). Both Mathematical Modeling (MM) and Computational Thinking are practices reflected in the most recent mathematics and science K-12 frameworks [3]. In addition, the Common Core State Standards [4] corroborates PISA's focus and suggests that mathematics programs should "prepare students to solve problems arising in everyday life, society, and the workplace (rich contextual problems)" [4] (p. 7). The design of rich contextual problems builds on content knowledge and also develops communication, collaboration, and problem-solving, which are the hallmarks of innovation in STEM. Connections between STEM disciplines are described in *Principles and Standards for School Mathematics*, prescribing that students "solve problems that arise in mathematics and other contexts" [5] (p. 402).

The current demand for developing 21st-century learners [6] emphasizes skills including communication, collaboration, creativity, and critical thinking, and include other skills on the OECD's list [2], such as research and inquiry, self-direction, initiative, persistence, information use, systems thinking, and reflection. Many educators embraced problem-based learning (PBL) environments and STEM lessons to offer elementary students the opportunity to develop these 21st-century skills. In addition, problem-based learning promotes collaborative problem-solving competencies by providing opportunities for an individual to effectively engage in a process where students attempt to solve a problem, by sharing the understanding and effort required to come to a solution and pooling their knowledge, skills, and efforts to reach that solution [2].

As a nation, we need to prepare a diverse workforce equipped with 21st-century skills that will succeed in the STEM jobs created by the data revolution and new technologies [7]. Framed in the context of problem-based learning and the demand to develop 21st century skills, our project aims to revive a community of engaged, collaborative STEM learners in underserved schools, through mathematical modeling.

2. Conceptual Background

2.1. Opportunities for Rigorous and Deeper Mathematics Learning for Diverse Students

Many groups across education utilize 'achievement gaps' when describing the persistent efforts at increasing representation of diverse people in the science and STEM fields. Describing these 'gaps' as 'opportunity gaps', accounts for inequitable access to resources, coursework, and experiences that exist for diverse students in schools across the U.S. [8]. Minority students have limited access to advanced coursework such as Calculus and Physics, which sets the foundation for entrance in college-level STEM bachelor programs [9]. This disparity begins at early childhood, within individual classrooms, across grade levels and schools within districts. Evidence is "compelling that children who are identified as Black, Latinx, Indigenous, language learners, poor and with disabilities along with other marginalized learners, do not have the same opportunities as their peers to access and learn in mathematically powerful spaces" [10] (p. 1).

The recent National Assessment of Educational Progress (NAEP) assessment, which assesses students at grade 4, 8, and 12, showed that 59 percent of grade 4 students were at or below the basic level, compared to 32 percent at the proficient level and 9 percent at the advanced level [11]. Based on this, we can say that less than a third of our grade 4 students are at the proficient levels, applying mathematical knowledge in real-world

situations, demonstrating conceptual understanding, and providing written solutions with supporting information explanations. Less than one in ten students are at the advanced levels, demonstrating their ability to solve complex and non-routine problems and justify their answers and solution processes. The assessment data results indicate that school mathematics needs to prioritize and support instructional practices that focus on deeper mathematical understanding, strategies, explanation, and justification. The question becomes, then, how can we begin to impact the experiences of diverse students before they enter middle and high school, so they develop the nature, skills, and attitudes involved in mathematical investigation and process skills, to reach advanced proficiency before their secondary school trajectories are set? What opportunities can we ensure for every student so that they can develop these critical mathematical skills and dispositions earlier in elementary grades? If we are serious about promoting mathematical literacy, as described above, and to broaden the participation of underrepresented youth, research on equitable teaching practices for mathematics in grades K-12 is critical. What changes are the mathematics education community ready to embrace to prepare our students for the future demands in our 21st century world?

2.2. Mathematical Modeling as a Vehicle for Catalyzing Change

Recently, the National Council of Teachers of Mathematics [10] issued three volumes entitled *Catalyzing Change, Initiating Critical Conversations*, with the following four key recommendations.

- (1) Broadening the purposes of learning math. Each and every student should develop mathematical understanding as confident and capable learners; understand and critique the world through mathematics; and experience the wonder, joy and beauty of mathematics.
- (2) Creating equitable structures in mathematics. Early childhood and elementary mathematics should dismantle inequitable structures, including ability grouping and tracking, and challenge spaces of marginality and privilege.
- (3) Implementing equitable mathematics instruction. Mathematics instruction should be consistent with research-informed and equitable teaching practices that nurture children's positive mathematical identities and strong sense of agency.
- (4) Developing deep mathematical understanding. Early childhood settings and elementary schools should build a strong foundation of deep mathematical understanding, emphasize reasoning and sensemaking, and ensure the highest quality mathematics education for each and every child.

The authors of the series note that traditionally, open-ended, complex problems are typically accessible to students considered to be gifted and "high ability" learners. At the same time, rote and procedural skills focus on students deemed "low ability" learners.

Select few in high achieving groups are afforded reasoning and problem-solving opportunities, those placed in low ability groups often receive a steady direction of remediation—practice repetition and reinforcement of basic facts and procedures [10] (p. 27).

Research shows that students from historically marginalized groups placed in less rigorous curricula have lower achievement in mathematics [12], which perpetuates opportunity gaps. Many researchers [13–15] stated that the significant disparity manifests in many forms, including resources, highly qualified teachers, access to cognitively rich, and relevant tasks and curriculum. In turn, these disparities impact mathematics learning outcomes and continue to preserve race, class, language, and ability hierarchies, further marginalizing learners from diverse populations. Collective action is needed to disseminate these essential practices and provide appropriate professional development for elementary teachers to introduce mathematical modeling as part of the curriculum and learning experiences for all students.

This project focused on better understanding the nature of equitable teaching practices in elementary mathematics classrooms and the potential of mathematical modeling instruction on promoting in-depth content knowledge and collaborative problem-solving. The

research on equitable teaching practices for STEM learning in elementary grades is critically important if we want to broaden the participation of underrepresented youth. According to researchers like Turner et al. [16], a vital consideration to broadening participation from diverse learners is to establish norms for participation by creating structures to position each student as a full participant in math and recognize that participation builds agency. These norms can help teachers position every student as competent mathematics thinkers and valued mathematical contributors. This positionality also has important implications for assigning competence and power in the mathematics classroom. Myers et al. [17] emphasize allowing all students to have voice and ensuring equitable ownership of ideas, encouraging justification, and engaging every student as a “creator of mathematical knowledge, recognizing what students already know, and self-empowering students by helping them see themselves as doers of mathematics” [17] (p. 22).

Mathematics modeling has the potential to “re-humanize” mathematics [18,19] by providing students opportunities to make connections, and experience mathematics in a way that can help them navigate the realities in their daily life. Mathematical Modeling has several attributes of an approach that “(re)humanizes” mathematics. By tapping into students’ funds of knowledge, mathematics becomes personally relevant [15], and using real-world problems illustrates the usefulness of mathematics to students’ everyday situations [20] and becomes a tool through which both students and teachers can affect social change [15]. Finally, we see mathematical modeling “(re)humanizes” mathematics by drawing on Bartell et al.’s (2017) Equitable Mathematics Teaching Practices—positioning students as capable, recognizing multiple forms of discourse and language as a resource, attending to students’ mathematical thinking, and building power and agency [21].

We need to capitalize on students’ funds of knowledge, the informal knowledge, wonder, and curiosities they bring to school [22–24], and the knowledge they acquire from their community or local context. In fact, in previous studies conducted by the project team, the context a teacher can draw from their immediate community might differ, based on their community, region, and culture. Capitalizing on this knowledge and leveraging local STEM contexts, like agriculture, renewable energy, and other local industries, can link familiar ideas to students’ scientific ideas. Recognizing and capitalizing on these local resources requires skill and deep content knowledge. Weak teaching in content areas can become a gatekeeper to STEM for schools, where teachers lack deep content knowledge. In many school districts, STEM lessons are implemented without much vetting for rich content or sound pedagogy, due to the lack of high-quality professional development for developing efficacy in knowledge and pedagogies associated with STEM instruction. While we know that the most effective professional development is rooted in experiences that allow teachers to dig into content, analyze instructional decision-making, reflect on their practices, and formulate responses [25,26], traditional grab-and-go, or workshop structures are still the norm. Our study focused on professional development using Lesson Study [27], with the teachers as co-designers. This design allows for meaningful professional development that engages teachers, researchers, and designers of modeling tasks relevant for elementary students, requiring in-depth knowledge of defining, and teaching mathematical modeling to students.

Mathematical modeling is an example of an interdisciplinary area that brings together mathematics and technology to solve real-world problems. Mathematical modeling shares several attributes of the operational definition for “Computational Thinking”. It requires a process of tackling open-ended problems, considering variables, and using decomposition, modeling, and algorithms [28]. In mathematical modeling, a model is often obtained or derived as a simplification of a real-world problem in a mathematical form. This form could be represented using equations, graphs, or tables, and one of the challenges is often to look for a solution that describes the trend in the data observed. In this process, it is often essential to solve the mathematical form produced using sophisticated techniques or technology. Mathematical modeling naturally offers classroom teachers proficient in mathematical content and technology a unique opportunity to take mathematics content

standards and infuse it with 21st-century skills to enhance student learning. Our increased dependence on technology combined with our society's complexity and interconnectedness has led to significant changes in the types of mathematical thinking required by our society [29]. The mathematical modeling approach differs from traditional word problems by providing students the opportunity to solve genuine problems and to construct significant mathematical ideas and processes, instead of merely applying previously taught procedures [29].

As stated in the Common Core Standards for Mathematical Modeling, "Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process [5] (p. 60)" These real-world problems tend to be messy and require multiple math concepts, a creative approach to math, and involve a cyclical process of revising and analyzing the model. Mathematical modeling (see Figure 1) translates between the real world and mathematics in both directions, defining the real world as everything that has to do with nature, society, or culture, including everyday life, school, and university subjects, or scientific and scholarly disciplines different from mathematics [30].

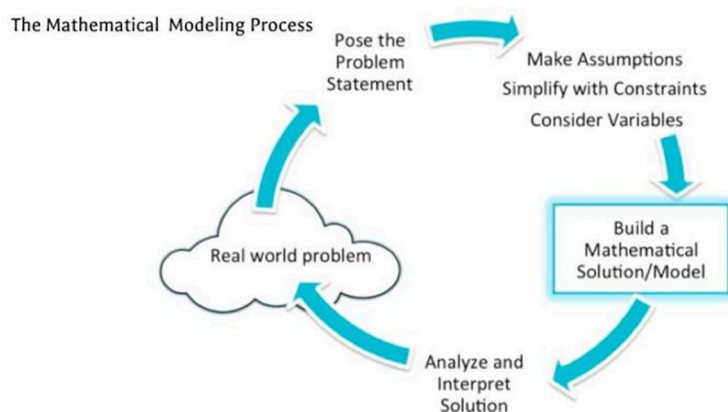


Figure 1. Math Modeling Process.

This modeling process is similar to the method used to define mathematical literacy, which includes formulating the problem, employing strategies, and interpreting the solution back to the problem. According to OECD's Program for International Student Assessment (PISA) framework, the relationship between mathematical reasoning and the problem solving (modeling) cycle is described using these processes [2] (p. 8).

In order for students to be mathematically literate they must first be able, to use their mathematics content knowledge to recognize the mathematical nature of a situation (problem), especially those situations encountered in the real world, and then to formulate it in mathematical terms. This transformation—from an ambiguous, messy, real-world situation to a well-defined mathematics problem—requires mathematical reasoning. Once the transformation is successfully made, the resulting mathematical problem needs to be solved using the mathematics concepts, algorithms, and procedures taught in schools. However, it might require the making of strategic decisions about the selection of those tools and the order of their application—this is also a manifestation of mathematical reasoning.

Finally, the PISA definition reminds us of the students' need to evaluate the mathematical solution by interpreting the results within the original real-world situation.

Over the past three decades, several studies worldwide also demonstrated that students who are appropriately guided to consider real-world situations have positive experiences with mathematical modeling [31–33]. For example, in a study involving fourth- and fifth-grade students who did not have any prior mathematical modeling experience from Switzerland, Belgium, and Japan, all students engaged in pairs of word problems [33]. The first of the pair included a straightforward word problem and the second was a nonroutine modeling problem. Students performed poorly on the modeling problems and tended

to solve them as if they were traditional word problems. However, when the teachers in Switzerland engaged students in critical thinking about the sufficiency of the information provided to get an answer and to consider what additional information would be needed, the percentage of students responding appropriately increased. In an Australian study with seven 4th-grade classes, students selected swimmers for the Olympic and Commonwealth Games. The content included ranking and aggregating data, calculating and ranking means, and creating and working with weighted scores. The study found that students developed far more advanced concepts than what would have been taught in a traditional classroom [34]. Such student experiences demonstrate the importance of both conceptual and procedural mathematical knowledge, a deeper focus on big ideas, usefulness outside of school, higher-order modeling process, and on research-based learning progressions [34].

In the following case study of a mathematical modeling task embedded in a community-based STEM problem, we engaged teachers in the modeling process, starting with situating the task within a real-world context. Teachers and students co-created the problem that required mathematical modeling.

3. Methods

This case study focuses on a team of teachers and students in a Lesson Study (LS) on a STEM unit focused on using mathematical modeling, to make significant decisions to design and plan for a sustainable edible garden in their community. The team included two second-grade teachers, Cindy and Lina, three fourth-grade teachers, Jackie, Jared, and Matt, and a mathematics coach, Emma. The second graders were in the age range of 7–8 years and the fourth graders were in the age range of 9–10 years. Cindy and Lina taught at one school where Emma was a coach and Jared and Matt taught together at another school. Jackie taught at a different school than all the others. The schools in this district had a high diversity in terms of ethnicity, socio economics levels, and linguistic and cultural backgrounds.

For the Lesson Study, teachers collaboratively planned for the School Garden Project. Teachers formed the following research focus—how do students take a social, environmental or local issue and translate it into a mathematical task? Their lesson focused on exploring how mathematics can appropriately take on important local issues for elementary age children. By looking to make the best use of each of their school's outdoor spaces, the Lesson Study team added another driving question for their project-based lesson—how might we take an environment friendly/conscious approach to improve our space. The teachers had a slightly different purpose for the space (i.e., Butterfly Garden, Future Courtyard, and Sustainable Community Edible Garden). As a team, we all observed the first iteration of the launch of the unit on the Butterfly Garden, through the research lesson in Cindy's second-grade classroom. Immediately following the lesson study, we debriefed as a team, discussing student learning, revisions to the lessons, and subsequent lessons that would follow the launch. After Cindy's research lesson, all other teachers, including the other second-grade teacher, Lina, and the other three fourth grade teachers, Jackie, Jared, and Matt enacted the lessons in their own respective classrooms and wrote lesson reflections. The fourth grade teachers geared up the mathematics focusing on concepts for their grade level. The unit took about five to seven days as it had a hands-on integrated science component. As a part of the final step of the Lesson Study, the team got together and synthesized their collective professional learning and outcome at the Lesson Study Symposium.

The two guiding research questions included:

- (1) How do elementary teachers develop students' capacity to engage in mathematical modeling while attending to equitable teaching practices?
- (2) How do teachers view teaching through mathematical modeling in terms of teaching and learning mathematics?

As researchers, we kept a researcher log and collected lesson artifacts (e.g., lesson plans, student work, transcripts of the lesson debrief, and lesson observation memos). For

the first research question, we analyzed the lesson reflections, transcripts of the lesson debriefs and lesson observation memos, using multiple cycles of process coding. Data collected from the study was loaded on Dedoose and coded using one of the methods called process coding [35], where one codes for gerunds, using -ing words, to code for actions taken by the teachers and students in the modeling process. More specifically, we coded for actions that teachers took to attend to students' equitable participation and how the teacher elicited students' mathematical thinking. First, we labeled segments of text into two broad categories of "planning" and "enacting" because we wanted to focus on the teachers' descriptions of their planning process and how they enacted the lesson. A second round of reviewing and consolidating codes allowed for us to narrow down distinct teaching practices that specifically focused on (a) supporting the mathematical modeling process and (b) promoting equitable student participation. We attached the artifacts specific to the tools and structures to teaching practices that supported these processes. We identified codes into categories and identified themes of equitable teaching practices along with tools and structure (see Table 1). For the second research question, we coded and categorized to pull broader themes from the teachers' reflections on their views about teaching, through mathematical modeling.

Table 1. Structures, Routines, and Tools Used to Promote Equitable Participation.

Mathematical Modeling Process	Structures, Routines, and Tools * for Equitable Participation
Problem Posing	<ul style="list-style-type: none"> • Brainstorming through Notice and Wonders • Math Happening Routines • Carousel walk to Make Thinking Public
Making Assumptions and Defining Variables	<ul style="list-style-type: none"> • Whole Group Think Time • Paired Think Pad- to deepen and share individual and partner's ideas
Build solution/Model	<ul style="list-style-type: none"> • Planned for Purposeful questions to make math visible • Math Tools: Graph paper, technology, calculators, information guides
Analyze and Interpret the solution and Connect it back to the Real World	<ul style="list-style-type: none"> • "Give One, Get One" Sticky Notes • Group Presentations with Gallery Display of Plans

* all described in detail in the narrative.

4. Results

The initial lesson conducted in a second-grade classroom as the first iteration of the Lesson Study was "The Launch", which sparked excitement in the students as they were introduced to the task as an authentic need in their school. The principal came in to announce that they were doing a makeover for their school garden. For that first lesson, the buy-in was clearly set for students as they started to wonder about all the elements that needed to be considered in the problem (the variables). After the lesson, the Lesson Study team debriefed that even though mathematical modeling has an element of "open-endedness", it was important for teachers to have structures, routines, and tools to help facilitate mathematical thinking and equitable participation. This became the deliberate focus of the following iteration of the lesson conducted by other members of the LS team and a focus of the first research question exploring how elementary teachers can develop students' capacity to engage in mathematical modeling, while attending to equitable teaching practices.

4.1. Teachers Actions for Attending to Equitable Participation when Eliciting Student Thinking

Across the second iteration of the lesson done in the other LS teachers' classrooms, we found several strategies and tools (Table 1) that teachers used to involve students in the modeling process, to promote equitable participation and to focus on mathematics. There were notable structures and tools that teachers planned for and used during the modeling process, to promote equitable participation and to generate mathematical thinking.

Below, we share examples from the lessons that marked innovative ways teachers engaged students in problem posing, making assumptions and defining variables, building solutions and models, and analyzing and interpreting the solution back to the real-world situation.

4.1.1. Valuing Students' Sense of Wonder and Excitement while Problem Posing

Teachers used different ways to create awe, wonder, and excitement around their mathematical modeling task. One routine called a "Math Happening" takes an event that is happening personally or in their community, and looks for ways mathematics can help them solve an issue or explain the phenomenon. At one of the schools, the principal came and shared a special message with the second graders, "We have this abandoned garden, and I need your help to create a sustainable butterfly garden" (See Figure 2). Not only did they take the invitation by the principal seriously, but as they were learning in science about the importance of butterflies as pollinators that strengthen the ecosystem, the second graders were committed to the project. The photo of an abandoned garden prompted students to wonder about all the possibilities they could make of the outdoor space.



Figure 2. Use of a photo of the abandoned garden to inspire their math modeling task.

One of the fourth-grade teachers, Jackie, used a school initiative, the Outdoor Sustainable Garden project, and described how she elicited multiple ideas from her students. Jackie documented in her reflective memo:

"To introduce the problem to students, I posed the question: Our school is looking to make the best use of our garden space. How might we take an environmentally friendly approach to improving our space? Students immediately had many ideas. Before I finished sharing the first slide, a student was thinking mathematically about solving the problem. To activate students' background knowledge about habitats, watersheds, ecosystems, and taking care of the environment, students did a carousel walk and added ideas to charts around the classroom. After this, we looked at photos of other school gardens and walked outside to have a fresh look at the garden. The whole group was enthusiastic and full of ideas about how they might modify the existing space."

In the excerpt, the teacher mentions a “carousel walk,” a structure to encourage equitable participation. Posting multiple charts around the room, students circulated around others’ posters to add their ideas. The teachers decided to use this tool to share more ideas and encourage equitable participation among students, in this process of brainstorming ideas for the space that they wanted to develop outdoors. As the students generated questions, the teacher charted the ways students were thinking about what makes the “best” community garden. Some descriptions of the “best” included a garden that had the most variety of plants, while others considered use of space or cost effectiveness as the criteria.

4.1.2. Positioning Students as Contributors in Making Assumptions and Defining Variables

As Jackie moved her students to the next phase of the modeling cycle, she prompted students to make assumptions and define variables to mathematize and constrain the problem to make the mathematics more tractable. She stated, “Before students began to work on their solutions, we defined the variables in the whole group setting by identifying things that can be changed and things that cannot be changed.”

To encourage mathematical discourse and idea generation, she used a tool called a “Think Pad”, a structure that supported students’ ideas being heard and considered, as students paired up to build on each other’s ideas. This structure also positioned each and every student as valuable contributors to the modeling process. The question that each pair thought about was, “How might you use math to build a garden? What do you know about community gardens? What are you wondering?” Each pair took turns sharing what they thought and wondered. After both members shared their answers, the “ThinkPad” routine asks the pair to write what they thought and wondered. The pair’s work in Figure 3 shows how they wondered about how much space they would have and what tools they would need.

Think Pad	
How might you use math to build a garden?	
What do you know about community gardens?	
What are you wondering?	
Partner A thinks and wonders...	Partner B thinks and wonders...
<ul style="list-style-type: none"> • what is the math we will use? • How big is the lot? • How big is the planting place? 	<ul style="list-style-type: none"> • will everybody have their own space to plant? • what will we plant? • How much space do we have to a plant?
We both think and wonder...	
<p>We both think How much space will have and what tools will we use!</p>	

Figure 3. Think Pad to build on partner’s idea.

By thinking about the mathematics that they might use, students started to think about important quantities. For example, Partner A asked, how big is the lot, while Partner B asked, will everybody have their own space to plant? In this exchange, one student asked about the whole area while the other student was interested in each student’s area.

4.1.3. Drawing on Students' Mathematical and Experiential Knowledge Bases while Building Solutions

To encourage collaboration, the teacher had students work in teams of four or five distributing tools like large grid paper, laptops, measurement tools, and school supplies. Students used their research skills to find the costs of items and other mathematical connections to the problem. One student searched for information on spacing for different plants and depth for seed planting. Another group of students was thinking about how to use the grid paper. At first, they did not think to measure the outdoor space but as they started drafting their design, they saw the need to measure in order to create a scale model on the grid paper. This allowed students to recognize the need to use measurement to solve their problem. The group decided that one inch on the paper would be equivalent to one foot in the garden. The teacher circulated among the groups in the classroom and monitored students as they were planning their garden and noted some of the dilemmas students encountered in this real and messy problem related to scaling their drawing.

It was interesting that there were only a few students who wanted to go back to the garden space and measure as they started to create their plan. The space is a long narrow garden along one side of the school building. After measuring the group decided to use two sheets of the grid paper to make their plan so they could include the entire garden space. One student in the group decided to calculate the area of an unplanted space to create a herb garden. She measured the perimeter of the space when she was outside, and then transferred her ideas to the grid paper. The shape was rectangular with a triangle-shaped extension. After she decided to have one square represent 12 inches on the grid paper, she wondered about how to find the area of the triangular space she had also measured on grid paper. This is a concept she did not have any experience with, but she thought that the hypotenuse had half squares and she could find the area by putting halves together to make wholes. As I watched this student's enthusiasm as she engaged with this problem, I saw the power of modeling mathematics. She used her background knowledge but also discovered new concepts.

In this excerpt, the teacher celebrates student's initiative to create a scale model and use the mathematics she knew when she encountered mathematics that was technically beyond the grade level objective (i.e., area of a triangle is a fifth/sixth-grade learning objective). The student used the grid paper to compose square units by combining the partial triangle units to find the total area.

More opportunities for mathematizing happened as students decided to add a pathway or sidewalk that engaged students in thinking about cost by unit rate.

Another group of students in my class was thinking about reasonable sidewalk width. They considered how many people should fit through the sidewalk and to leave enough space for the plants so that they did not grow over the walkway. These students used meter sticks and acted out the problem by standing side-by-side. Next, they researched the cost of building a sidewalk and observed that "concrete is really cheap" after finding an article online quoting \$5.25 per square foot. To determine how many square feet were in their newly expanded sidewalk plan, a student made a box around herself with meter sticks and considered how to think about square foot blocks contained within the space.

In reflecting on how mathematical modeling afford multiple mathematics concepts to connect through the problem authentically, Jackie commented:

"The student reflections really showed the power of teaching math through mathematical modeling. Our research question prior to launching the task was whether students would naturally find the math in the real-world problem and they did. They mathematized the problem in a variety of ways. The following day during the morning meeting, students shared what math they used while working on the garden problem. I heard ideas such as addition, multiplication, measurement, decimals and money. I observed students using estimation and considering what is reasonable as they worked through the problem."

She points out how students became aware of all the mathematics that came into play "naturally", while they worked on this modeling task. Jackie continued in her reflection

that the teacher's role was critical in facilitating the work with purposeful questions to keep the mathematics focused and encourage peers to ask questions of each other to orient them towards the mathematics, as they refined their solution and their model.

Questioning was a powerful tool to redirect students toward a more mathematical line of thinking if needed. The student groups also worked collaboratively, and organically divided parts of the task, based on interest, as they worked to solve the problem. When students shared their solutions, other classmates asked thoughtful questions about the groups' choices and probed for possible revisions to their solutions.

4.1.4. Using Multiple Forms of Discourse and Representations to Analyze and Interpret the Solution to Connect Back to the Real World

The final phase allowed students to present their model and display, to allow them to communicate their mathematical ideas. In the Sustainable Garden case, their model included a drawing and some calculation that included area for each of the plant types, and budget for the cost of soil, plants, and other materials. As students presented their plan, the teacher employed a structure called "Give one, get one," where they give one comment and get one from every student. This routine allowed students to practice their skills in celebrating as well as critiquing each model. Using sticky notes, the students would leave comments for each of the models with either noticings, connections, or questions. The structured discourse process prompted students to bring their analytical and critical thinking skills, as they posed questions for their peers.

This final phase was necessary for the young modeler because through the "give-one, get-one," students would get a question from a peer asking about an assumption they made or a variable they did not consider. Using a Gallery Display (see Figure 4), students had the opportunity to respond to their peers with questions, and discuss how they might refine their solution or model.



Figure 4. Gallery display with presenters pitching their sustainable garden plan.

It also empowered students to understand that communicating one's math with drawings, words, and numbers allowed others to entertain their ideas. The teacher was intentional in using this share to elevate students' ideas, making sure that all solutions were valued and evaluated for the strength in students' thinking. To ensure accountability in the group work, the teacher assigned roles of recording the groups' collective thinking and reporting them, so that all members were important contributors of mathematical ideas.

4.2. Teachers' Views of Teaching and Learning through Mathematical Modeling

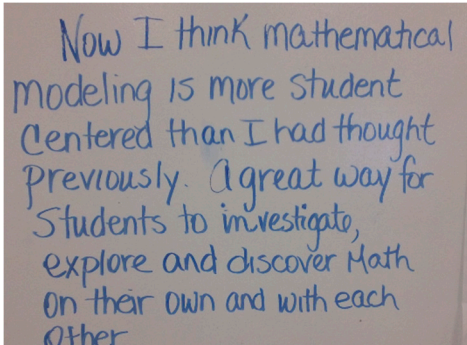
To address the second research question, we analyzed teachers' reflections, lesson debriefs, and interviews to look for themes on how they viewed the approach to teaching through mathematical modeling. There were three major themes that we found across comments after the lesson debriefs, interviews, and teachers' reflective memos—(1) integrating important mathematics concepts while giving students ownership of the mathematic; (2) assessing for both content and 21st century process skills through mathematical modeling; and (3) changes in the way teachers viewed mathematics teaching and learning.

4.2.1. Integrating Important Mathematics Concepts while Giving Student Ownership of the Mathematics

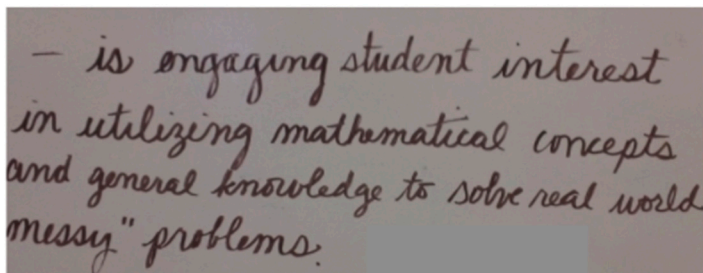
Mathematical modeling provided students with opportunities to bring in their lived experiences and multiple knowledge bases to tackle the problem. This gave students a reason for the mathematics that they employed and brought ownership to their mathematical learning.

The students really enjoyed the opportunity to use their whole brain to help tackle a real issue at our school. The students were given opportunities to integrate all subject areas and tap into their critical and creative thinking skills through this lesson. It is safe to say that none of the students were asked to approach math in this manner. It gave our young second graders ownership for their school community and the learning that they accomplished through the task. While talking through the learning that had occurred, the students were amazed at how many topics they were able to cover through just one project. Needless to say, the students had fun! (Cindy, a second-grade teacher's memo).

Teachers reflected using a visible thinking prompt called, "I used to think . . . now I think" (see Figure 5). Many of the changes in their thinking revolved around seeing mathematical modeling as a vehicle to create more student-centered learning, "driving their own learning through authentic real-life situations," and "connecting interdisciplinary concepts" and "higher-level thinking."



Now I think mathematical modeling is more student centered than I had thought previously. A great way for students to investigate, explore and discover math on their own and with each other.



- is engaging student interest in utilizing mathematical concepts and general knowledge to solve real world messy problems.

Figure 5. Teachers' reflection on "I used to think...Now I think math modeling is . . .".

There were many mathematical productive pathways that the teachers anticipated for the sustainable garden project. Teachers planning for the mathematical modeling lesson expressed appreciation for the "open-ended-ness." This open-ended, complex, and real problem provided the potential and power to engage students in many mathematically productive pathways. Below (see Table 2) are some classroom-tested ideas of how this group of teachers anticipated and planned for various mathematical opportunities and considered revisiting this task to add to the sophistication of ideas developed throughout the academic year.

Table 2. Mathematical Pathways for Planning for a Sustainable Garden.

Mathematical Pathway 1 (Optimization)	Maximizing the area for a garden plot You have some chicken wire to fence your garden so the rabbit won't eat all your veggies. What is the maximum size for your garden?
Mathematical Pathway 2 (Descriptive Modeling)	Garden area: This task challenges a student to use understanding of area and count squares to find the area of shapes on a grid. https://www.insidemathematics.org/sites/default/files/materials/garden%20design.pdf Drawing to Scale: A Garden: Use scale drawings to plan a garden layout using proportional reasoning and metric units. http://map.mathshell.org/lessons.php?unit=7310&collection=8&redir=1
Mathematical Pathway 3 (Predictive Modeling)	Plants need Space to grow Based on the data about space plants need to grow, design a garden plot of 12 by 12 feet; Use coordinate grids to locate using ordered pairs and create a legend with different plants http://extension.illinois.edu/firstgarden/planning/dictionary/veggies/index.cfm
Mathematical Pathway 4 (Descriptive and Predictive Modeling)	Budget your project Keep track of your expenses and minimize waste of supplies. How might you design a garden with a total cost of \$50? \$100? Create a proposal to secure money for planting a garden—request for seeds, tools, and other materials required to execute students' plan.
Mathematical Pathway 5 (Scientific modeling)	Data analysis-science connection When preparing to plant the space, germinate some seeds indoors and plant others of the same plant directly outside. Collect data about plant growth over time to compare this variable.

Preparing these potential fruitful mathematical pathways provided teachers more confidence to support students when their mathematical wonderings led them into a different path from other groups. Teachers also felt that the ownership of student group choice on the mathematical pathways they pursued allowed students to find purpose. These multiple pathways also allowed for natural differentiations, with opportunities for the teacher to tailor a problem for students with different mathematical backgrounds, without resorting to a modified, remedial task. Every member of the class, regardless of differing mathematics backgrounds, engaged in the sustainable garden project.

4.2.2. Assessing for Both Content and 21st Century Process Skills through Mathematical Modeling

Teachers commented that unlike some of their lessons that focus on a single mathematics objective, the mathematical modeling lessons naturally led them to teachable moments and formative assessment opportunities using several related skills, like computations, measurement, and budgeting. In addition, teachers' reflection revealed how mathematical modeling required using necessary 21st century process skills and an opportunity to see them in use.

In our lesson, we felt the students used the following mathematical skills—computation, non-standard, and standard measurement, drawing a model to scale and money. In our lesson, we felt the students used the following creative and critical thinking skills—questioning; flexibility and collaboration; visualization; point of view; and decision-making.

Students learned the necessity of creating a grid plan to represent the square footage of the lot and allocation of different plants. As student groups planned their sustainable garden, team members negotiated decisions, and students utilized their research skills to defend which plants to plant in the garden. In addition to multiple mathematics skills, this mathematical modeling task required some research around planning for a sustainable garden.

In one of Jackie's memos, she wrote how students were researching what plants to purchase and how much space each plant needed to grow properly. Once they found information about plant spacing, students had to adjust their plans to accommodate for more area. Some student groups had to change their plant choice plan as they learned more about how deer can eat all the harvest and added basil and lavender, learning that these

were deer-resistant herbs. Through this project, the word “sustainable” took on personal meaning for the students. One student group worked on the frequency of watering plants and decided to create a watering plan. Another student group worked on arranging their plants based on height so that all the plants could get sunlight.

The teachers created an anecdotal chart to record student thinking called “Mathematical Modeling Look-fors”. Five categories reflected the iterative phases of the MM process. To focus on the solution-finding and analysis of the model phase, teachers listed three essential skills in getting a solution—students making connections to prior knowledge, generating preliminary answers, and assessing the reasonableness of solutions. As students moved to the analysis phase, teachers again assessed modeling competencies using the MM Look for Checklist. Figure 6 shows a sample excerpt of a formative assessment tool.

Getting a Solution

Connections to prior knowledge	<ul style="list-style-type: none"> Organized plants tallest to smallest knowing that some plants grow so tall that it blocks sunlight for smaller plants used info from seed packets to determine how to space
Generate preliminary answers	<ul style="list-style-type: none"> Placed the tomato plant that was the tallest in the middle and considered space for other plants. Calculated fencing to enclose the plants
Reasonableness of solutions	<ul style="list-style-type: none"> planting guide data to select possible plants considering space allowed Decided to use a grid to make sure the plants would have more space.

Figure 6. Using an assessment grid to make note of the group’s solution method.

Mathematical discourse is one way students can express their mathematical proficiency. A discursive learning environment highly prizes verbal and written communication. As teachers who work with a high population of English language learners, it was critical for them to support students with limited English to participate in the modeling process. Many structures that they had in place and described in the early sections, namely, partner talk, carousel walks, “give one, get one”, allowed students to engage in a variety of conversation settings to practice the language of mathematics with peers in small and large group structures. Additionally, teachers created sentence stems to support initiating mathematical conversations focused around the modeling thinking processes. As a team, we charted these sentence stems and had them posted on sentence strips so that students had a way to come into any one of the conversations (see Figure 7). The questions under each phase were written in the first person, to prompt students to use their metacognitive skills, “what information do I need?” The sentence frame, “If I know _____, then I can figure out _____” was written to support their thinking about important quantities and assumptions that helped them mathematize the problem.

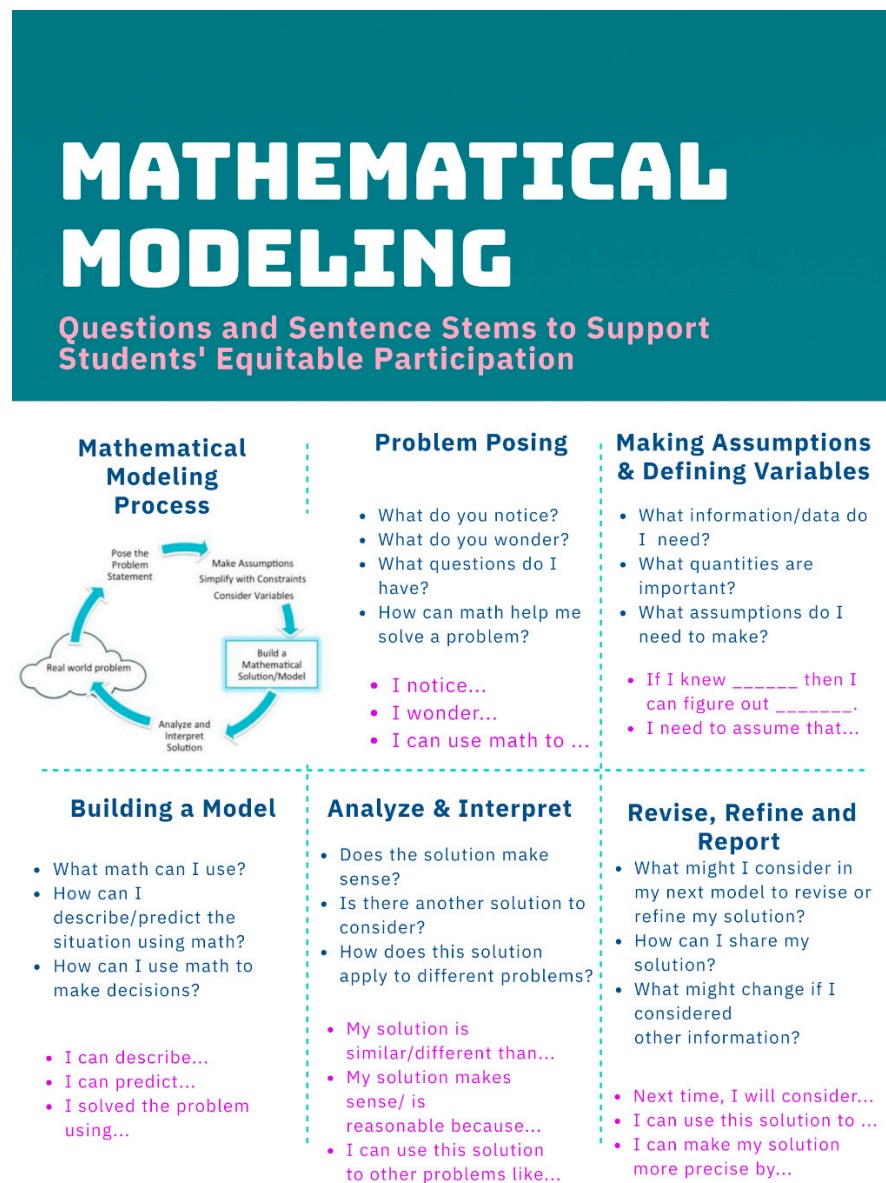


Figure 7. Question and sentence stems to promote “Mathematical Modeling Talk”.

4.2.3. Changes in the Way Teachers Viewed Mathematics Teaching and Learning

We analyzed teachers’ reflections to examine how MM changed their views of teaching and learning mathematics. For this theme, we shared the transformation of three of the teachers in the group.

Matthew was a novice but avid and enthusiastic teacher-designer, who appreciated the creative process of mathematical modeling. Matthew had four years of teaching experience at the time of the study. At the beginning of the school year, Matthew said he “... really wanted to revamp what math looked like for myself and for my students. So, coming in the first day of school we did a mathematical modeling project on setting up the classroom—that was day one.” He continued MM with his fourth-graders, completing a different mathematical modeling task for each unit covered during the school year. The class did a MM project on the future courtyard of their school that was under renovation.

Matthew noted that at the beginning of the year, he had to be explicit with and talk about each step of the MM process. As the year progressed, however, he saw his students gain confidence with the MM process. “I let them go and just kind of saw that they wanted to be more fluid with it. And that’s how they really think too, in class... [The MM process] is more just kind of a web thinking.” Matthew attributed part of his success with MM

to his mindset, saying, "I've always really been open-minded with teaching. I have a really strong growth mindset, so it was easier for me to let my kids go and see what they can do." Matthew summarized the key attributes of mathematical modeling with the following statement:

"The philosophy of mathematical modeling, is keeping it open-ended... pull out the math in the task the students are working on...I wanted to keep it open-ended and we've kept it open-ended, with the students, by letting them choose what they could do with the overarching task. So, we made sure that the driving question and that the task itself was open-ended enough so that students could use their creativity, and kind of see what they were able to do with the math instead of us giving it to them."

Jared was a reticent but thoughtful adopter of mathematical modeling who found his "comfort zone". A career switcher, Jared had three years of teaching experience at the time of the study. Jared taught at the same school as Matthew and was drawn to MM by Matthew's enthusiasm. He reflected that he did six MM projects during the year "because Matthew was very gung-ho from the beginning." Like Matthew, Jared jumped into MM at the beginning of the year, doing an MM project for each unit with his 4th grade class. Reflecting on his mindset at the outset, he said

"I was like, you're going to be doing a lot, I'm going to have to step up to the plate so . . . I think the idea and starting it [MM] off went pretty well in terms of my kind of motivation and intrigue."

Jared's level of comfort in initiating and implementing MM grew across the year. Initially, he took up the MM innovation because of Matthew's confidence and belief that their students could do MM. Looking back across the year, however, Jared saw the value of MM himself.

"But, in terms of teaching math modeling, I mean I think I've really embraced it; I enjoy it. I think it just works so much better from the teacher perspective because it's not day after day giving them a focus lesson, it's putting them in the front seat of doing math. And that's what they need to do because you can teach them until they're blue in the face how to do this, this, and this but until they can do it on their own, they're not going to feel comfortable with it."

As the year progressed, he commented that he became comfortable redirecting his students, allowing them to struggle with mathematics themselves and giving them enough but not too much guidance. He saw his students as coming a long way in terms of feeling more in charge of their learning and saw that they grew in their confidence in mathematics.

"Then with math modeling I think they were able to explore the various ways they can do it or it was much more student centered where someone sitting here would teach someone else this is how I did it or this is how I do it. And they grew to accept each other knowing that there's more than one way to do it. And before I would teach them the various ways and let them choose, but I think now they feel so much more confident that they can do it on their own."

Cindy was an experienced teacher rejuvenated by seeing her students eager to learn. Cindy, a second- grade teacher reflected on how mathematical modeling "ignited" learning and engaged even the reluctant learners in her classroom. She stated, "It was clear that mathematical modeling ignited a high level of engagement in the learning process for all students. Even students that Lina and I were initially concerned about remained on task and interested in the activity." She also expressed how she learned that students did not need so much front-loading of direct instruction before beginning on the mathematical modeling task; instead, she comments:

"We learned the importance of allowing the students to just begin the task without feeling the pressure to front load so much of the learning for them. It was surprising that as soon as we stepped away, the students took the math task in ways that we could have never planned."

She shared how she changed her view of mathematics after having planned and implemented a mathematical modeling lesson, as she shared:

“The process of planning and implementing the lesson has changed the way I think about math. I wasn’t challenging my students to find their own answers, and quite often I was giving too much support. Now I allow my students more opportunities to work through their math without me. While I tried to plan for every bump in the road, as teachers, we know children are unpredictable.”

This statement clearly showed how Mathematical Modeling encourages teachers to switch ownership of the mathematics to the students, giving them more agency as they challenge their students to “find their own answers.”

I understand now that students need specific tools to navigate through this increasingly complex world. Mathematical Modeling provides these critical skills by exposing students to approachable, everyday scenarios, to recognize the underlying mathematics in these scenarios, and to understand how the application of mathematics can be applied to real-life situations. Today’s learner is exposed to quite a bit of scientific information. A student’s ability to comprehend and apply this information requires number sense, computation, statistics, probability, and percentages. We need to prepare our students for further studies in math and science. Mathematical Modeling answers the student’s question of “Why do I need to learn this?”, regardless of their future profession.

5. Discussion

As teachers engage students in taking more ownership of mathematical modeling, the process also helps them design better tasks that help to carefully orchestrate modeling discussions. In such facilitation, teachers often attempt to balance multiple instructional goals as they navigate their daily instruction, ranging from curricular, mathematical, pedagogical, and personal goals. Explicit focus on individual components, particularly thinking about the real world, sense-making, elaborating students’ mathematics, making assumptions, and formalizing curricular topics, allows them to fulfill these instructional goals. Modeling also enhances their pedagogical practices. Teachers integrate approaches that help students develop fluency in working with different representations and models to develop their conceptual competence or understanding through modeling.

Our paper connects mathematical modeling with the four recommendations from NCTM, highlighting its potential to advance these goals and act as a catalyst for change that mathematics educators should consider in early elementary mathematics education. In terms of broadening the purposes of learning mathematics [10], students who engaged in mathematical modeling experienced the wonder and joy of solving a problem to improve their environment, which broadens the purpose of learning mathematics. They felt a sense of agency as they worked on improving their school community garden space. Each student engaged in the task to contribute to their class plan for the sustainable garden and had a chance to confidently share their plans and critique and refine ideas of other mathematicians in the classroom. Mathematical modeling aims to provide students with community-based tasks that they can connect to personally, and make meaning of the mathematics within the world they are living in.

In terms of creating equitable structures in mathematics [10], the community garden’s familiar and relevant context gave every student entry to the problem. It leveled the playing field and did not privilege only some students. Students who were learning English and who had special learning needs were not pulled out of class for separate instruction but immersed in the task with peers who complemented one another with the multiple knowledge bases that each student brought. The task piqued students’ interest because it was set in their community and generated collaboration and meaningful discourse, while students designed their solution. The classroom communities built through mathematical modeling embodied high expectations, and empowered students as the holders of mathematical knowledge.

In terms of implementing equitable mathematics instruction [10], teachers created participation structures with partner talk, carousel walks, “give one, get one,” and sentence stems, to nurture students’ positive mathematical identities and a strong sense of agency.

By having thought partners with their peers, students had a mathematical community behind them, as they dared to take risks, challenge each other's ideas, and persevere in finding a solution. The mathematical modeling structure poses the teacher with the crucial role of facilitating the learning that is owned by the students. As teachers work in the background to further push the groups of students in their thinking, every student's mathematical ideas take the forefront of the instruction, as they help their peers tackle a meaningful challenge.

Finally, in terms of developing deep mathematical understanding [10], the elementary students used the mathematics that they learned to describe the plan for the garden design and consider the budget to purchase plants and soil. They had realistic constraints and variables to consider with the garden lot and a budget to provide fruitful complexity to work through with their mathematics skills. As showcased in this case study, the realistic and messy nature of the modeling task set in the real world allows students to encounter mathematical ideas many times beyond the grade level objective. Ultimately, students are enticed to engage in more rigorous mathematics. In our project, we heard students say, "Teach us the math so we can solve this problem!", the empowered disposition that we want to cultivate in every student.

As mathematics educators and classroom teachers "purposefully work to transform early childhood and elementary classrooms into mathematically powerful spaces [10] (p. 1)", we need to make these spaces engaging with authentic and meaningful mathematical work that develop students' positive mathematical identities and a sense of agency. Progress in mathematics education will have to reflect our complex and ever-changing world to develop every student's mathematical literacy. We believe introducing mathematical modeling in earlier grades would make this commitment a reality and bring positive changes in the mathematics learning opportunities and outcomes for every child.

6. Future Directions

Mathematical modeling typically used in grades 9–12 and beyond is only beginning to gain traction in elementary education [36]. In the United States, the Common Core State Standards in Mathematics [5] Practice 4, Model with Mathematics, is intended to cut across grades K-12. As such, MM should be a part of elementary students' earliest mathematics experiences and not an approach to be used if there is spare time or as an afterthought to traditional instruction.

We acknowledge that teaching through mathematical modeling is one of the most ambitious reform-oriented initiatives. It is ambitious because it aspires to teach all students academic content and have them apply their knowledge to solve authentic real-world problems [37]. It requires students to move through a process of mathematizing a real-world and using mathematics to solve a problem. Ambitious teaching requires teachers to be responsive to what students do, as they engage in problem-solving performances, and hold students accountable to learning goals that include procedural fluency, strategic competence, adaptive reasoning, and productive dispositions [37].

One future direction for our research team is to better understand the nature of the required specialized knowledge needed by teachers when enacting mathematical modeling, which we call Mathematical Modeling Teaching Competencies, which include knowledge of the modeling process, curriculum, pedagogy and most importantly students. Teachers in our Lesson Study leveraged the strengths of MM with its authenticity and real-worldness of the problems, to engage their students. They were able to identify engaging contexts, build on student ideas, use authentic problems, and make real-life connections for their students. We found that those teachers who successfully designed and implemented MM in their classrooms knew their curriculum deeply, both horizontally within their grade level and vertically across different grade levels. Furthermore, these teachers were able to take advantage of mathematical opportunities arising in the MM process, recognizing the potential of aligning opportunities with their curriculum. Teaching is multidimensional in that it involves creating a learning environment, orchestrating

participation from students, and eliciting and understanding students' mathematical ideas drawn from the mathematical tasks. This multifaceted and complex nature of teaching requires teachers to know about their students' mathematical thinking to listen to their ideas and capitalize on the opportunity to engage them around emerging mathematical ideas. As we continue our research, we look to expand the field's knowledge in ways to support teachers' mathematical modeling competencies so that each and every learner can have the experience of learning through authentic problems to see the useful and powerful nature of mathematics.

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