

Article

# Fuzzy Stochastic Automation Model for Decision Support in the Process Inter-Budgetary Regulation

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**Abstract:** The purpose of this article is to study the theoretical foundations of the concept of fiscal decentralization, as the main path of self-development of the national economy of any country, and to develop mathematical tools that support decision-making in the aspect of “hard” budget constraints. The study of the problems of fiscal policy formation in foreign countries presented in modern scientific literature has revealed that the degree of application of the concepts of “soft” and “hard” budget restrictions is an actual topic in the theory of fiscal federalism. It has been substantiated that decision-making within the framework of “soft” budget constraints (financial assistance) leads to low tax autonomy of territories and limited liability of regional and municipal authorities for the results of their financial policy. As a research hypothesis, we put forward the thesis that it is necessary to create conditions for encouraging subnational authorities to support the territorial economy by granting them the possibility to use part of the taxes collected in the respective territories. The implementation of this thesis has given rise to the problem of quantifying decisions made regarding the establishment of standards for the distribution of tax revenues between budgets of different levels of the hierarchy of the country’s budget system. In terms of solving this problem, the author has constructed mathematical models based on the use of synthesis of mathematical apparatus of the theory of stochastic automata, fuzzy algebra, and simulation. In terms of solving this problem, the author proposed the use of mathematical modeling methods. The article presents the results of constructing economic and mathematical models to support decision-making in the vertical distribution of tax revenues between budgets. The models include stochastic automata, as mathematical abstractions, describing the expedient behavior of an economic agent when choosing management alternatives for territories of different levels of economic development. The transition functions of automaton models are formally described on the basis of the synthesis of mathematical apparatus of the theories of stochastic automata operating in random environments and fuzzy sets. The expediency property of the behavior of automaton models is justified by proving the corresponding theorems. The random environment in which stochastic automata are immersed is formed by a simulation model. The article demonstrates the results of experiments carried out on models, as well as a conceptual scheme of interaction between the automaton and simulation models.



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## 1. Introduction

Economic growth is the main goal of each state’s economic policy, which is confirmed by a wide range of scientific publications on this topic [1–4]. Due to the fact that the level of development of the national economy is determined by the aggregate economic potential of its subnational territories, creating conditions for their economic growth is a key problem of any country’s macroeconomic policy. The analysis of scientific works on the study of the economic state of foreign countries has demonstrated the characteristic feature of subnational territories revealed in these studies, which includes the differentiation of the level of their socio-economic development. The problem of economic inequality is currently an important determinant that negatively affects economic activity, which is

reflected in a number of studies. Modern research on this topic includes the works of M. Brachert, E. Dettmann, M. Titze [5]; S. Achten, C. Lessmann [6], L. Muinelo-Gallo, O. Roca-Sagalés [7], Achten, C. Lessmann [8]. The acuteness of the problem of growing processes of spatial inequality within countries is argued by a significant undermining of the national economy. Moreover, as confirmed in the article by T. F. Remington [9], the following paradox is characteristic of economic processes occurring in heterogeneous countries. It is paradoxical, according to T. F. Remington [9], that according to economic theory, the initial income inequality between sub-national regions of the country should go into interregional convergence, but in countries such as Russia and China, this transition did not happen. In the Russian Federation [10–13], there are processes of increasing differences in the level of economic development of subnational territories, causing many social problems. Understanding international economic research in the context of spatial heterogeneity (A. Todes, I. Turok [14], T. Kennedy [15], F. Venturini [16]) allowed us to conclude that it is very important to solve the problems of strengthening the economic systems of municipalities for the formation of economic growth policy for the entire state. Thus, disproportions in the socio-economic development of territories are a global problem, and in order to achieve uniformity of the economic space, any state needs to use tools that affect economic processes. One of the most effective levers of influence on the economy is fiscal policy based on the principles of federalism. The theory of fiscal federalism assigns fiscal decentralization a leading role in encouraging subnational governments to develop the economy in their territories in order to accelerate economic growth. The effectiveness of decentralization is proved by the theorem of W. Oates [17,18]. The impact of budget system decentralization on the quality of public administration [19,20] and economic growth [21–26] is discussed and argued by the results of numerous empirical studies. At the same time, territorial self-development becomes a modern paradigm for the evolution of administrative-territorial units of any country. In accordance with this paradigm, territories should aim for economic growth by increasing the efficiency of their use and increasing their internal resources. In this context, a necessary condition is budget self-development, defined as the ability of an administrative-territorial entity not only to provide all areas of its economic activity with tax revenues but also to promote sustainable, balanced development by increasing these revenues. As proved by many researchers [27–33], transfers from the state budget reduce fiscal incentives. Another feature of Federal systems is that in some countries, Federal and territorial tax bases are jointly owned [32]. Such a problem in the aspect of financial relations exists in Russia. In the Russian Federation, tax revenues are mainly concentrated at the Federal level. The results of research on the trajectories of movement of tax revenues to the territorial budgets of Russia have shown a tendency to increase their revenues to the consolidated budget of the Russian Federation. Thus, in 2016, 2017, 2018, the Federal budget was credited with 47.4%, to 49.2% and 52.3% of all revenues of the consolidated budget of the Russian Federation, while in the consolidated budgets of RF, subjects received only 36.6%, 35%, and 32% of total revenues. Established not only in Russia but in other countries, the system has asymmetric fiscal relations, that is, the growing vertical imbalances resulting in the use of “soft” budget constraints, an alignment level of budgetary security of territories through financial assistance from the Federal budget [33–36]. The main drawback of counter cash flows in the form of transfer infusions is a decrease in the incentives of regional and municipal authorities to increase the tax potential [25,30,37,38]. This way of solving the problem of equalizing the level of fiscal security of subnational territories threatens the stability of the national economy, not to mention ensuring economic growth. One of the key problems of applying the strategy of fiscal decentralization as a catalyst for economic development in the model of fiscal federalism is to increase the revenue autonomy of the budgets of subnational territories by redistributing joint tax revenues between the Federal center and subnational territories. The solution to this problem involves the formation of a system of norms for the distribution of tax revenues collected in a given territory between the budgets of various hierarchical levels of the administrative-territorial structure. An effective way to solve this problem is

to develop and use mathematical models that allow you to give a quantitative justification for the decisions made and assess their consequences.

The purpose of the article is to develop an interdisciplinary approach to creating adaptive mathematical models to support decision-making in the distribution of tax revenues between budgets along the vertical line of power, as well as to have mathematical models built into the information system that formally describe the appropriate behavior of the decision-making subject.

This article contributes to the discussion on the formation of fiscal policy in the model of fiscal federalism and also gives impetus to research on the development and use of adaptive, intelligent mathematical models to support decision-making in the problem of fiscal decentralization, focused on tight budget constraints. As a leading method for creating a mathematical model, the article proposes a synthesis of the mathematical apparatus of the theory of stochastic automata operating in random environments [39] and fuzzy algebra [40]. The mathematical model of the fuzzy automaton developed in the article formally describes the appropriate behavior of the subject making a decision when distributing tax revenues between the budgets of the higher and lower levels of the budget system of a Federal state. The expediency of the behavior of the constructed fuzzy automaton is mathematically strictly justified by proving the corresponding theorems given in the article. The article is organized as follows. The first section substantiates the relevance of research in the field of fiscal decentralization by building mathematical models. Section 2 analyzes research on the concept of tight budget constraints, as well as research in the field of automatic modeling of inter-budget relations. Section 3 describes the problem of constructing a fuzzy automaton for decision support in the vertical distribution of tax revenues between budgets. Analytical expressions are obtained for the final probabilities of the automaton choosing its states. The state of the automaton is interpreted as a share of deductions from a specific tax to the lower-level budget. In the same section, the behavioral aspects of the constructed fuzzy automaton are investigated by proving theorems about its expediency of behavior. The constructed automaton mathematical model functions when interacting with a simulation model that forms a random environment in which the fuzzy automaton is immersed (Section 4). The random environment is formed by generating possible values of budget revenues and expenditures by the simulation model. In the same section, the scheme of computer implementation of the automatic model functioning as part of information technology is given. The results of experiments on the constructed model are demonstrated.

## 2. Literature Review

### 2.1. Research Areas on the Concept of Tight Budget Constraints

Currently, there is a steady global trend towards decentralization. The impact of fiscal decentralization on economic growth has been widely discussed in the scientific literature of the late last century. Literature analysis has shown that decentralization takes different forms in different countries [41]. Research on the impact of fiscal policy as an instrument of economic stabilization on the state of the economic system in different countries was conducted by such researchers as Lindbeck A [42], Aaron H., McGuire M. [43], Snyder V. V. [44], and Green K. V. [45]. Studying the effects of intergovernmental subsidies was the subject of the research of Schwallie D. [46], Romer T., Rosenthal H. [47]. The General conclusion of these works is that it is useful in some situations to apply the concept of not only hard but also soft budget restrictions. Currently, economists are very interested in the form of China's fiscal policy due to the high average annual growth rate of its economy. The reforms carried out in this country based on the territorial principle have led to a high degree of decentralization in regional areas and, as a result, to economic growth [46,48,49].

Some modern authors also reveal the negative consequences of decentralization. Issues related to soft budget constraints were investigated by Roden J., Eskeland G. S., Litvak J. [50]; and Raiden J. [51–53]. After analyzing the behavior of the subnational governments of Argentina, Brazil, Canada, China, Germany, Hungary, India, Norway, South Africa,

Ukraine, and the United States, the authors conclude that decentralization, in particular soft budget constraints, can sometimes lead to large budget deficits and large macroeconomic problems. Studying Brazil as the most decentralized country in the developing world, the author J. Roden [53,54] revealed that the Brazilian States, in comparison with the provinces of many developing countries, receive a significant part of their income from taxation. Noting some negative features of decentralization, J. Rodden [53,54] concludes that large and stable aggregate deficits occur when subnational governments are simultaneously dependent on intergovernmental transfers and free to borrow. Vertical budget imbalances and the impact of fiscal decentralization on the quality of public administration are analyzed by Eyraud L., Lusinyan L. [55], Hindricks J., Lockwood B. [56]. The authors argue that fiscal centralization reduces the degree of electoral discipline. The problems of applying “soft” and “hard” budget restrictions were discussed in articles by Chulkov D. [57], Hopland A. O. [58], Bethlendi A., Lentner C., and Nagy L. [59]. In the unitary Scandinavian countries, the concept of “administrative federalism” is used [60], in which a decentralized government is created from the center, and local authorities form an integral part of the public sector and are agents of this center. However, in Turkey, after granting some freedom to local authorities, the positive effects could not be improved, and the authorities carried out a reform, as a result of which local authorities became more dependent on inter-budget transfers [61]. There are many other studies that are based on considerations of the effectiveness of fiscal decentralization [62–66]. Taking into account the non-linearity of the outcomes of various forms of fiscal policy, modern works consider the issues of quantitative assessment of informed decision-making in this area. Based on empirical research, many works are devoted to identifying systemic relationships between economic phenomena [67–70]. The use of methods of economic and mathematical modeling is of great importance in the study of the feasibility of using fiscal decentralization in the development of tools to support decision-making in this regard. However, a study of modern publications on the development of economic and mathematical models has revealed a certain vacuum in the field of describing the appropriate behavior of subjects in the decision-making process under fiscal decentralization. The formalization of decision-making processes in view of the complexity of the latter requires interdisciplinary research. This article offers a synthesis of mathematical abstractions of the stochastic automaton, fuzzy-multiple and simulation models for decision support, as well as risk assessment in the aspect of implementing strict budget constraints.

## *2.2. Analysis of Recent Research in the Field of Automatic Simulation of Inter-Budgetary Relations*

In the modern scientific literature, the results of research are presented, mainly related to “soft budget constraints” (SBS—Soft Budget Constraint). SBS refers to the effect in the economy in which decision-makers rely on external assistance in the event of an insolvency risk. At the same time, mathematical models were proposed for determining the volume of various transfer infusions that perform the functions of equalizing the level of budget security of administrative-territorial entities. Along with the positive effect of these models, it should be noted that they lack the ability to help increase the interest of subnational authorities in increasing the tax base of their territories. The author of this article suggests shifting the focus to the use of the concept of “hard” budget restrictions in the process of inter-budget regulation. In accordance with this concept, subnational authorities are given the opportunity to use part of the taxes collected in the territory under their jurisdiction. This makes it possible not only to ensure a balanced budget but also to improve the quality of this balance by increasing the share of their own tax revenues, while increasing the responsibility of the authorities to support the economy at the proper level. The concept of “hard” budget restrictions was introduced into scientific circulation in 1979 by J. Kornai [71,72], a Hungarian economist, and is due to the need to increase the motivation of the authorities in strengthening economic activities leading to economic growth. At the same time, the subjects of choice of management decisions assume all management risks of losses associated with fluctuations in the economic environment.

Currently, there is a global trend of applying the concept of “hard” budget constraints, leading to decentralization in financial management [73]. The role of decentralization in the management of public goods, as the most important factor in ensuring economic growth, is logically strictly justified by the theorem of W. E. Oates [18,19]. Adhering to this concept, the article sets the task of creating economic and mathematical tools to support management decision-making to establish standards for the distribution of tax revenues between territorial budgets along the vertical line of power. An analysis of procedures related to the implementation of the incentive function of inter-budget regulation has shown that, at present, the standards for the distribution of tax deductions are set on the basis of discretionary decisions that are taken autonomously by subnational governments. The methodological, technological, and methodological vacuum in the development of clear procedures and algorithms in this area creates favorable conditions for subnational governments to apply the concept of “soft” budget restrictions associated with the use of inter-budget regulation tools such as inter-budget transfers. This is inconsistent with the market-preserving model of fiscal federalism and contradicts global trends in fiscal decentralization. However, along with this, publications devoted to the construction of economic and mathematical models that use the concept of “hard” budget constraints in managing inter-budget relations began to appear in the scientific literature. An example is an adaptive learning model based on the construction of a stochastic automaton with linear tactics [74,75]. The automatic model proposed in E. Streltsova [74,75], which has selective tactics, formalizes the appropriate behavior of a decision-maker when setting standards for the distribution of tax revenues between a sub-Federation and a sub-region. The expediency property of the automaton behavior is mathematically strictly justified by proving the theorem [74,75]. However, the proposed model suffers from a number of disadvantages. First, the States of the automaton are integrated values that reflect various combinations of values of the distribution proportions of various taxes involved in the process of budget regulation. For a two-level system of local budgets, this model is not very effective, since in this case a differentiated approach is needed to select tax revenues of each type, which serve as levers of influence of local governments on the size of the tax base. Second, the structure of a stochastic automaton is such that in the case of a fine, this automaton can only move to neighboring condition, which limits research on the choice of tax distribution proportions in budget regulation. As a result, the obtained expressions for the final probabilities of the automaton’s stay in each of the conditions do not reflect all the possibilities of variation out of the amounts of deductions from taxes in the order of budget regulation. However, the approach of using such a mathematical abstraction as a stochastic automaton is promising, since it allows you to build adaptive models that describe the appropriate behavior of the subject of decision-making. Subsequently, this approach was further developed in the construction of an improved automaton model that formalizes the behavior of an economic agent in the process of choosing alternatives when implementing an active component of inter-budget regulation [75,76]. In this model, the automaton is able to make a transition from each of its States to any other state. As the States of the automaton, the values of the shares of deductions to the budgets of the sub-region from taxes intended for crediting to the budgets of higher levels of the budget system are taken. The proposed mathematical model in the form of an improved stochastic automaton lacks the above disadvantages and was used in information technologies of inter-budget regulation processes. Revealing the disadvantages of the transformed automaton model (the same probability of transition of the automaton from state to state with a penalty) in this article offers a more perfect model—the model of a fuzzy automaton operating in a random environment.

### 3. Materials and Methods

#### 3.1. Concept and Tools

In this section, the task is to build a mathematical model to support decision-making when determining the size of the standards for the distribution of tax revenues between

budgets of different levels of the hierarchy based on the application of the mathematical apparatus of the theory of stochastic automata operating in random environments.

In accordance with the theory of stochastic automata [39], the mathematical model of the automaton *AVT* proposed in this article is given by the following tuple  $AVT = (Inp, Ex, Con, F, \Phi)$ , where  $Inp = \{p, q\}$ ,  $p = \{p_1, p_2, \dots, p_m\}$ ,  $q = \{q_1, q_2, \dots, q_m\}$  is a set of input signals;  $Ex = \{e_1, e_2, \dots, e_m\}$ —a set of output signals;  $Con = \{con_1, con_2, \dots, con_m\}$ —a set of states;  $F : Inp \times Con \rightarrow Con$ —a function of the transition of the automaton from state to state;  $\Phi : Inp \times Con \rightarrow Ex$ —an output function. As states  $con_i, i = \overline{1, m}$ , the automaton takes the values of the share of deductions from tax to the budget of the sub-region in the process of implementing the active component of inter-budgetary regulation. States  $con_i$  are determined by dividing the segment  $[0, 1]$  into  $m$  equal parts. At the same time  $con_1 = \frac{1}{m}, con_2 = \frac{2}{m}, \dots, con_m = 1$  [77]. The automaton operates in a random environment, which is formed by the random nature of cash receipts to the budget and their expenditures from the budget. The values of budget revenues and expenditures are considered as perturbations. The values of budget revenues and expenditures are considered as perturbations. The outputs  $e_i, i = \overline{1, m}$  are the values of the budget's cash balances, which are affected by the state values  $con_i, i = \overline{1, m}$ . Due to the fact that the machine *AVT* operates in a random environment, its outputs  $e_i, i = \overline{1, m}$  affect it. The random environment reacts to the effects  $con_i, i = \overline{1, m}$  of the automaton *AVT* and sends signals to its input  $p_i, q_i, i = \overline{1, m}$ . The components  $p_i, q_i$  are interpreted as the probabilities of “non-penalty” and “penalty” in the states  $con_i, i = \overline{1, m}$  [76,77]. The input of the machine *AVT* receives a signal  $q_i$  that means the probabilities of “penalty” if the impact  $con_i$  led to the formation of the amount of cash balances in the budget  $e_i < 0$ . If a value is formed  $con_i$  at the output of the automate *AVT* as a result of exposure  $e_i > 0$ , then a signal  $p_i = 1 - q_i$  meaning the probabilities of “no penalty” is sent to its input. Thus, the values  $p_i, q_i$  are defined as the probability of the current surplus and deficit in the budget [76,77]. In accordance with [76,77], the automaton passes from one state  $con_i$  to another  $con_j, i \neq j$  when a signal is received at its input  $q_i$ . When a signal is received at the input of the automaton *AVT*, the automaton does not leave its state.

### 3.2. Synthesis of Automatic and Fuzzy-Multiple Approaches in the Decision Support Model

Due to the fact that a differentiated approach to the choice of budget regulation strategy should be applied to administrative-territorial units of different levels of economic development, the article suggests the decomposition of territories into two classes. The first class (Devel) is proposed to include administrative-territorial entities with increased ability to self-development at the expense of their internal resources. The second class (Anavel) includes territories that do not have this quality.

This classification requires an analysis of the socio-economic development of local administrative divisions, which is a complex task. According to the authors A. Luczak and M. A. Just [1], “there is no standard procedure for solving this problem at different levels of management” [1]. Such an analysis requires the development of a system of methods, indicators and techniques that adequately assess the features of the evolution of territories at different levels of administrative structure. The systems of indicators used by decision-makers or experts to assess the level of socio-economic development of a territorial unit depend on the professional knowledge of specialists, on the scenarios of possible developments developed by them, as well as on the specifics of the territory's economy. In the roles of indicators of evolution can be included both quantitative parameters (budget deficits, revenues, and expenditures; the volume of gross regional or municipal product per capita; assessment of production potential; levels of profitability of the main sectors of the economy, etc.) and qualitatively defined characteristics, which include many different institutional, environmental, and other indicators. Solving this problem requires the use of multicriteria decision analysis (MCDA) methods. A large number of scientific studies and works are devoted to the assessment of the economic condition of economic objects. Currently, various variants of fuzzy TOPSIS methods are widely used for solving multi-criteria problems of analyzing various objects. Among the

modern works devoted to the study of the economic development of state structures, the article by A. Luczak, M. A. Just [1] aroused the greatest interest. Authors A. Luczak and M. A. Just proposed original methods for assessing the level of economic development at the national, subnational and sub-regional levels of Poland using the MCDA procedure based on the tail selection method.

However, this article does not set the task of developing a method for decomposing local territorial units into groups based on their ability to self-develop. Due to the complexity of approaches and the large volume, this problem will be considered in other works by the author. The purpose of this article is to build a mathematical model within the framework of implementing the strategy of “hard” budget restrictions to support decision-making regarding the establishment of the share of splitting joint taxes between the budgets of the Federal and sub-regional levels, as well as to link this model with the level of socio-economic development of local territorial units.

For territories with high abilities for self-development, i.e., belonging to the class *Devel*, in the process of budgetary control is appropriate budget transfers to use the tool deductions from taxes, as the amounts of taxes collected are able not only to balance the regional budget but also enhance the interest of local authorities to strengthen and develop economic activities. For territories that do not have this property, i.e., belong to the *Andevel* class, it is less appropriate to use only tax deductions as a tool for inter-budget regulation due to the inability of these deductions to increase the level of budget security. In this case, it is appropriate to achieve a balanced budget by combining tax deductions with transfer payments.

In this regard, for the territory of each class, the article suggests the use of a qualitatively expressed measure of the expediency of establishing specific shares of deductions from taxes.

This expediency measure is a linguistic variable  $MEASURE = \langle T(MEASURE), U, M \rangle$  with a term set  $T(MEASURE) = \{High, Low\} \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ . The set components  $T(MEASURE)$  are the names of fuzzy sets that qualitatively describe the measure of expediency of assigning a high *High* and low *Low* level tax deduction standard for a given territory. The universe  $U$  is represented by the segment  $[0, 1]$ , on which the set of membership functions  $M = \{\mu_H, \mu_L\}$  of fuzzy sets *High* and *Low* is given:  $\mu^H : \{Con_i\}_{i=1}^m \rightarrow [0, 1]$ ,  $\mu^L : \{Con_i\}_{i=1}^m \rightarrow [0, 1]$ . The membership functions have a triangular form  $\mu^H, \mu^L$  and are described by the equations:

$$\mu^L = \begin{cases} 0, Con_i < 0; \\ 1 - Con_i, 0 < Con_i < 1 \\ 0, Con_i > 1; \end{cases} \quad \mu^H = \begin{cases} 0, Con_i < 0; \\ Con_i - 0, 0 < Con_i < 1 \\ 0, Con_i > 1; \end{cases} \quad (1)$$

Graphs of the functions  $\mu^H : \{Con_i\}_{i=1}^m \rightarrow [0, 1]$  and  $\mu^L : \{Con_i\}_{i=1}^m \rightarrow [0, 1]$  are shown in Figure 1. Based on this, the automaton *AVT* is represented by a two-component construction  $AVT = (A^L, A^H)$ , where  $A^L$  and  $A^H$  describes the behavior of the subject making a decision on the amount of standards for deductions to the budget of the territory with a low and high level of self-development, respectively.

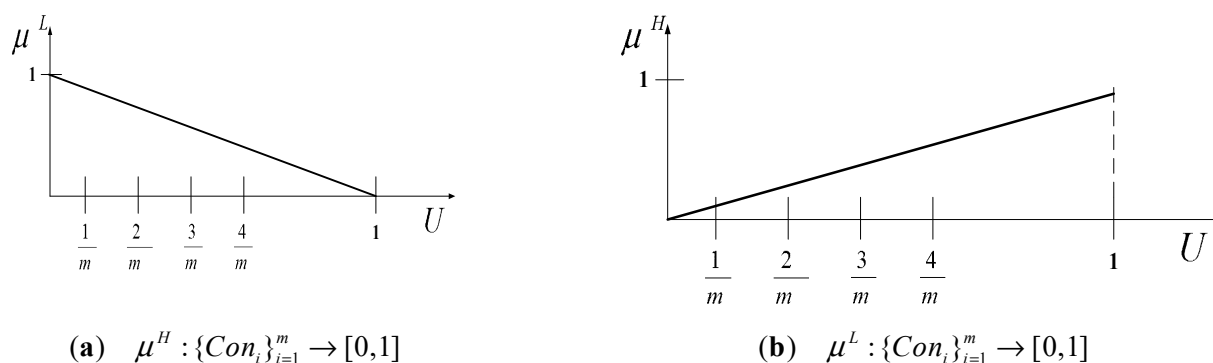


Figure 1. Membership functions of fuzzy sets High (a) and Low (b).

According to the theory of stochastic automata in random environments [39], the change of states of automata  $A^L$  and  $A^H$  is described by two functions of their transitions from state to state. When their input receives a “non-penalty” signal indicated by a variable  $p_i$ , the automata do not leave their States, and their transition functions are described by matrices

$$\|x_{ij}^H\| = \|x_{ij}^L\| = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 \end{pmatrix} \tag{2}$$

When the input of the automata are  $A^L$  and  $A^H$ , the “penalty” signal is received, and their functions of state-to-state transitions are described by matrices, respectively,  $\|y_{ij}^L\|$  and  $\|y_{ij}^H\|$ :

$$\|y_{ij}^L\| = \begin{pmatrix} 0 & \frac{m-2}{m} & \frac{m-3}{m} & \dots & \frac{m-m}{m} \\ \frac{m-1}{m} & 0 & \frac{m-3}{m} & \dots & \frac{m-m}{m} \\ \frac{m-1}{m} & \frac{m-2}{m} & 0 & \dots & \frac{m-m}{m} \\ \dots & \dots & \dots & \dots & \dots \\ \frac{m-1}{m} & \frac{m-2}{m} & \frac{m-3}{m} & \dots & 0 \end{pmatrix} \quad \|y_{ij}^H\| = \begin{pmatrix} 0 & \frac{2}{m} & \frac{3}{m} & \dots & \frac{m}{m} \\ \frac{1}{m} & 0 & \frac{3}{m} & \dots & \frac{m}{m} \\ \frac{1}{m} & \frac{2}{m} & 0 & \dots & \frac{m}{m} \\ \dots & \dots & \dots & \dots & \dots \\ \frac{1}{m} & \frac{2}{m} & \frac{3}{m} & \dots & 0 \end{pmatrix}.$$

Elements of matrices  $\|y_{ij}^L\|$  and  $\|y_{ij}^H\|$  are defined based on triangular membership functions  $\mu^L : \{Con_i\}_{i=1}^m \rightarrow [0,1]$  and  $\mu^H : \{Con_i\}_{i=1}^m \rightarrow [0,1]$ . Elements of transition matrices  $\|p_{ij}^L\|$  and  $\|p_{ij}^H\|$  automata  $A^L$  and  $A^H$  regardless of the input signal are determined based on the expressions  $p_{ij}^L = x_{ij}^L p_i + y_{ij}^L q_i$ ;  $p_{ij}^H = x_{ij}^H p_i + y_{ij}^H q_i$ . Matrices  $\|p_{ij}^L\|$  and  $\|p_{ij}^H\|$  are as follows

$$p_{ij}^L = \begin{pmatrix} p_1 & \frac{m-2}{m} q_2 & \frac{m-3}{m} q_3 & \dots & \frac{m-m}{m} q_k \\ \frac{m-1}{m} q_1 & p_2 & \frac{m-3}{m} q_3 & \dots & \frac{m-m}{m} q_k \\ \frac{m-1}{m} q_1 & \frac{m-2}{m} q_2 & p_3 & \dots & \frac{m-m}{m} q_k \\ \dots & \dots & \dots & \dots & \dots \\ \frac{m-1}{m} q_1 & \frac{m-2}{m} q_2 & \frac{m-3}{m} q_3 & \dots & p_k \end{pmatrix} \tag{3}$$

$$p_{ij}^H = \begin{pmatrix} p_1 & \frac{2}{m} q_1 & \frac{3}{m} q_2 & \dots & \frac{m}{m} q_k \\ \frac{1}{m} q_1 & p_2 & \frac{3}{m} q_3 & \dots & \frac{m}{m} q_k \\ \frac{1}{m} q_1 & \frac{2}{m} q_2 & p_3 & \dots & \frac{m}{m} q_k \\ \dots & \dots & \dots & \dots & \dots \\ \frac{1}{m} q_1 & \frac{2}{m} q_2 & \frac{3}{m} q_3 & \dots & p_k \end{pmatrix} \tag{4}$$

The values of the final probabilities  $\Phi^L = \{\Phi_1^L, \Phi_2^L, \dots, \Phi_m^L\}$ ,  $\Phi^H = \{\Phi_1^H, \Phi_2^H, \dots, \Phi_m^H\}$  of states of the automata  $A^L$  and  $A^H$  are determined from the expression  $\Phi^L = \Phi^L(T) \bullet \|p_{ij}^L\|$ ,  $\Phi^H = \Phi^H(T) \bullet \|p_{ij}^H\|$  where  $\Phi^L(T)$ ,  $\Phi^H(T)$ — transpose of the matrix. Analytical expressions for the final probabilities  $\Phi^L(T)$ ,  $\Phi^H(T)$  are shown in Table 1.



**Table 1.** Analytical expressions for final probabilities  $\Phi^L = \{\Phi_1^L, \Phi_2^L, \dots, \Phi_m^L\}$  and  $\Phi^H = \{\Phi_1^H, \Phi_2^H, \dots, \Phi_m^H\}$ .

Condition	Automatic $A^L$ The Final Probability	Automatic $A^H$ The Final Probability
$con_1 = \frac{1}{m}$	$\Phi_1^L = \frac{1}{q_1(2k-1) \sum_{i=1}^k \frac{1}{q_i(2k-i)}}$	$\Phi_1^H = \frac{1}{q_1(k+1) \sum_{i=1}^k \frac{1}{q_i(k+i)}}$
$con_2 = \frac{2}{m}$	$\Phi_2^L = \frac{1}{q_2(2k-2) \sum_{i=1}^k \frac{1}{q_i(2k-i)}}$	$\Phi_2^H = \frac{1}{q_2(k+2) \sum_{i=1}^k \frac{1}{q_i(k+i)}}$
...	...	...
$con_m = \frac{m}{m}$	$\Phi_m^L = \frac{1}{q_k(2k-k) \sum_{i=1}^k \frac{1}{q_i(2k-i)}}$	$\Phi_m^H = \frac{1}{q_k(k+k) \sum_{i=1}^k \frac{1}{q_i(k+i)}}$

The economic interpretation of the final probabilities  $\Phi_i^L \in \Phi^L$  and  $\Phi_i^H \in \Phi^H, i = \overline{1, m}$  is that they give a quantitative assessment of the feasibility of making decisions about the shares of deductions  $con_i, i = \overline{1, m}$  from a specific tax.

The next section of the article provides a proof of theorems about the expediency of automata behavior  $A^L$  and  $A^H$  the proposed construction.

### 3.3. Theorems about the Appropriateness of Behavior of Fuzzy Automata

In accordance with the theory of stochastic automata operating in random environments [39], a stochastic automaton behaves appropriately if its mathematical expectation of winning is greater than the mathematical expectation of winning an automaton choosing its States is equally probable. We prove that these properties hold for automata  $A^L$  and  $A^H$ .

**Theorem 1.** *The automaton  $A^L$  has the property of expediency of behavior. Evidence.*

In accordance with the theory of stochastic automata [39], a condition for the appropriateness of behavior is the following inequality is satisfied:  $M(A^L) > M_0, M(A^L) = \sum_{i=1}^m \Phi_i^L p_i$  where is the mathematical expectation of a win machine  $A^L$ ;  $M_0 = \frac{\sum_{i=1}^k p_i}{k} = \frac{\sum_{i=1}^k (1-q_i)}{k} = \sum_{i=1}^k \frac{1}{k} - \sum_{i=1}^k \frac{q_i}{k}$  – mathematical expectation of winning the machine, producing their actions are equally probable. Let us write an analytical expression for  $M(A^L)$ , substituting the values  $\Phi_i^L$  from Table 1 into it:

$$\begin{aligned}
 M(A^L) &= \frac{1 - q_1}{q_1(2k - 1) \sum_{i=1}^k \frac{1}{q_i(2k-i)}} + \frac{1 - q_2}{q_2(2k - 2) \sum_{i=1}^k \frac{1}{q_i(2k-i)}} + \\
 &+ \frac{1 - q_3}{q_3(2k - 3) \sum_{i=1}^k \frac{1}{q_i(2k-i)}} + \dots + \frac{1 - q_k}{q_k(2k - k) \sum_{i=1}^k \frac{1}{q_i(2k-i)}} = \\
 &= \frac{1}{\sum_{i=1}^k \frac{1}{q_i(2k-i)}} \left( \frac{1 - q_1}{q_1(2k - 1)} + \frac{1 - q_2}{q_1(2k - 2)} + \frac{1 - q_3}{q_1(2k - 3)} + \dots + \frac{1 - q_k}{q_k(2k - k)} \right) = \\
 &= \frac{\sum_{i=1}^k \frac{1 - q_i}{q_i(2k-i)}}{\sum_{i=1}^k \frac{1}{q_i(2k-i)}} = \frac{\sum_{i=1}^k \frac{1}{q_i(2k-i)} - \sum_{i=1}^k \frac{q_i}{q_i(2k-i)}}{\sum_{i=1}^k \frac{1}{q_i(2k-i)}} = \frac{\sum_{i=1}^k \frac{1}{q_i(2k-i)} - \sum_{i=1}^k \frac{1}{(2k-i)}}{\sum_{i=1}^k \frac{1}{q_i(2k-i)}} = 1 - \frac{\sum_{i=1}^k \frac{1}{(2k-i)}}{\sum_{i=1}^k \frac{1}{q_i(2k-i)}}
 \end{aligned}$$

We need to prove that  $1 - \frac{\sum_{i=1}^k \frac{1}{(2k-i)}}{\sum_{i=1}^k \frac{1}{q_i(2k-i)}} > \sum_{i=1}^k \frac{1}{k} - \sum_{i=1}^k \frac{q_i}{k}$ , or  $\frac{\sum_{i=1}^k \frac{1}{(2k-i)}}{\sum_{i=1}^k \frac{1}{q_i(2k-i)}} < \sum_{i=1}^k \frac{q_i}{k}$ .

Considering the left side of the inequality  $\frac{\sum_{i=1}^k \frac{1}{(2k-i)}}{\sum_{i=1}^k \frac{1}{q_i(2k-i)}}$ , we introduce the notation  $\frac{1}{2k-i} = \omega_i$ . In accordance with this notation, the left side of the inequality will have the form of the weighted average harmonic  $\frac{\sum_{i=1}^k \omega_i}{\sum_{i=1}^k \frac{1}{q_i} \cdot \omega_i}$  with weights  $\omega_i = \frac{1}{2k-i}, i = \overline{1, k}$ . As you know, the weighted average harmonic is less than the weighted arithmetic average:

$$\frac{\sum_{i=1}^k \omega_i}{\sum_{i=1}^k \frac{1}{q_i} \cdot \omega_i} < \frac{\sum_{i=1}^k q_i \cdot \omega_i}{\sum_{i=1}^k \omega_i}.$$

In our case, a series of numbers  $\frac{1}{2k-i}, i = \overline{1, k}$  is ascending, and a number of numbers  $q_i$  is decreasing due to the fact that the larger the share of deductions from the tax, the less likely the deficit (i.e., the penalty of the machine). However, the larger the weights have small values of options, the smaller the value of the weighted average, so you can write

$$\frac{\sum_{i=1}^k q_i \cdot \omega_i}{\sum_{i=1}^k \omega_i} < \frac{\sum_{i=1}^k q_i}{k}.$$

By virtue of transitivity, we have  $\frac{\sum_{i=1}^k \omega_i}{\sum_{i=1}^k \frac{1}{q_i} \cdot \omega_i} < \frac{\sum_{i=1}^k q_i}{k}$ , which was required to prove.

We prove a theorem on the expediency of the behavior of the automaton  $A^H$ .

**Theorem 2.** A stochastic automaton  $A^H$  has the property of expediency of behavior. Evidence.

Similarly to the proof of the previous theorem, we will use the condition of expediency of stochastic automata, derived in [39]:  $M(A^H) > M_0$ , where  $M(A^H) = \sum_{i=1}^m \Phi_i^P p_i$ —the mathematical expectation of winning a stochastic automaton  $A^H$ ;  $M_0 = \frac{\sum_{i=1}^k p_i}{k} = \frac{\sum_{i=1}^k (1-q_i)}{k} = \sum_{i=1}^k \frac{1}{k} - \sum_{i=1}^k \frac{q_i}{k}$ —the mathematical expectation of winning an automaton that performs its actions is equally probable. We write an analytical expression for  $M(A^H)$ , substituting in it the values of the final probabilities  $\Phi_i^H$  of the automaton being in its states from Table 1:

$$M(A^H) = \frac{q_1}{q_1(k+1) \sum_{i=1}^k \frac{1}{q_i(k+i)}} + \frac{q_2}{q_2(k+2) \sum_{i=1}^k \frac{1}{q_i(k+i)}} + \frac{q_3}{q_3(k+3) \sum_{i=1}^k \frac{1}{q_i(k+i)}} + \dots + \frac{q_k}{q_k(k+k) \sum_{i=1}^k \frac{1}{q_i(k+i)}}$$

As a result of some transformations, we get:

$$M(A^H) = \frac{1}{\sum_{i=1}^k \frac{1}{q_i(k+i)}} \left( \frac{1}{k+1} + \frac{1}{k+2} + \frac{1}{k+3} + \dots + \frac{1}{k+k} \right) = \frac{\sum_{i=1}^k \frac{1}{(k+i)}}{\sum_{i=1}^k \frac{1}{(k+i)} \cdot \frac{1}{q_i}}.$$

In the last expression, we introduce a notation  $\rho_i = \frac{1}{(k+i)}$ , according to which the mathematical expectation of winning a stochastic automaton  $A^H$  will take the form

$$M(A^H) = \frac{\sum_{i=1}^k \rho}{\sum_{i=1}^k \rho_i \cdot \frac{1}{q_i}}.$$

The last expression confirms that the mathematical expectation of

winning  $M(A^H)$  a stochastic automaton is a weighted harmonic mean with weights  $\rho_i$ ,  $i = \overline{1, k}$ . However, the weighted harmonic mean is less than the weighted arithmetic mean:

$$\frac{\sum_{i=1}^k \rho}{\sum_{i=1}^k \rho_i \cdot \frac{1}{q_i}} < \frac{\sum_{i=1}^k \rho_i \cdot q_i}{\sum_{i=1}^k \rho_i}.$$

Consider the value of the weighted arithmetic mean, with respect to

which it is known that  $\frac{\sum_{i=1}^k \rho_i \cdot q_i}{\sum_{i=1}^k \rho_i} < \frac{\sum_{i=1}^k q_i}{r}$  if small values  $q_i$  of quantities have small weights.

Then the following inequality holds:  $\frac{\sum_{i=1}^k \rho_i}{\sum_{i=1}^k \rho_i \cdot \frac{1}{q_i}} < \frac{\sum_{i=1}^k \rho_i \cdot q_i}{\sum_{i=1}^k \rho_i} < \frac{\sum_{i=1}^k q_i}{r}$ . Due to transitivity, it is

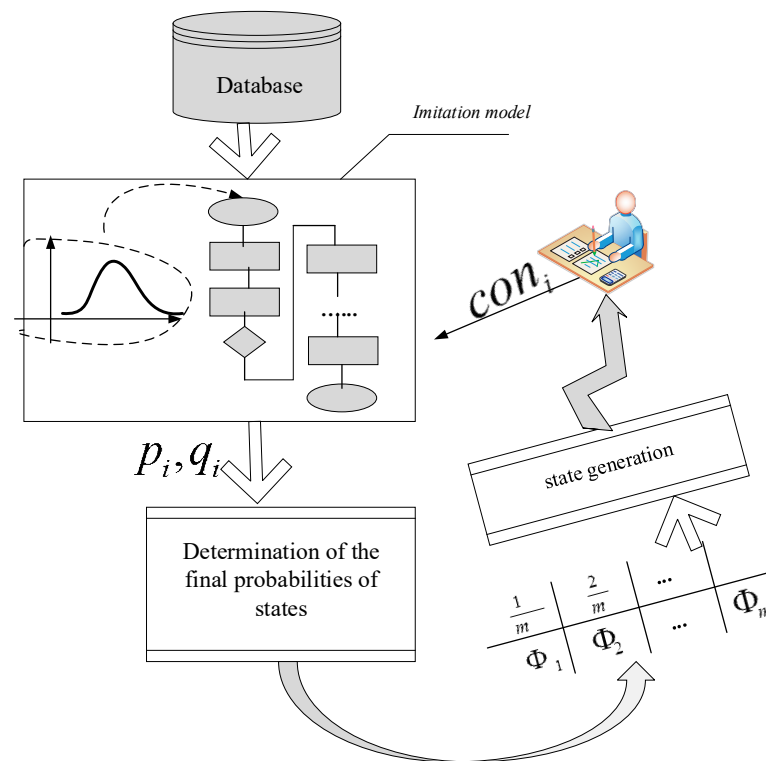
possible to write  $\frac{\sum_{i=1}^k \rho_i}{\sum_{i=1}^k \rho_i \cdot \frac{1}{q_i}} < \frac{\sum_{i=1}^k q_i}{r}$ , which was required to prove.

The economic interpretation of the expediency of the behavior of automaton models is that with an increase in memory capacity (memory capacity refers to the number of States), the automaton will win more often and lose less often. Due to the fact that under winning (in the model this signal “non-penalty”) and losing (in the model this signal is “penalty”) the automatic in the condition  $con_i, i = \overline{1, m}$  accepted values  $p_i, q_i$ , meaning respectively the probabilities of surplus and deficit budget, the constructed automaton model aimed to balance the budget, which are deductions from joint taxes.

#### 4. Computer Implementation and Experimental Results

##### 4.1. Information Technology to Support Decision-Making in the Shared Distribution of Tax Revenues

Analytical expressions for final probabilities  $\Phi_i^L, \Phi_i^H, i = \overline{1, m}$  contain values  $p_i, q_i, i = \overline{1, m}$ , that fix the reaction of the random environment to automata  $A^L, A^H$  and are interpreted as “no penalty” and “penalty”. These values reflect the risks of decisions made regarding the share of tax revenue distribution between budgets. As defined earlier, machines are fined if there is a probability of a current budget deficit. To determine the values  $p_i, q_i, i = \overline{1, m}$  the author proposed a simulation model, in interaction with which the researcher determines the share of deductions to the budget from the tax (Figure 2). A formal description of the simulation model is given in the author’s early works [77]. For a better understanding of the author’s idea, the article provides a conceptual representation of this model. The input control variables of the simulation model are the state values of the automaton  $con_i, i = \overline{1, m}$ . In the economic sense, the states  $con_i$  of the automata are interpreted as the shares of deductions to the budget of a lower level of the budget system from tax, which is intended to be credited to the budget of a higher level.



**Figure 2.** Interaction of automatic and simulation models in the implementation of the incentive function of inter-budget regulation.

As perturbations, we consider the values of various types of current tax and non-tax budget revenues, as well as the values of current expenditures from the budget. The output signals of the simulation model are values  $p_i, q_i, i = \overline{1, m}$  that reflect estimates of the probabilities of a budget surplus and deficit at different values of values  $con_i, i = \overline{1, m}$  (in the automaton model, they are considered as estimates of the probabilities of rewards and punishments of the automaton). The values of perturbations are formally described by the laws of probability distribution, which are used to generate their possible values using the method of statistical tests. By varying the values  $con_i, i = \overline{1, m}$  for a specific type of tax, the economic agent evaluates them by analyzing the output signals  $p_i, q_i, i = \overline{1, m}$ . It is assumed that automatic and simulation models are embedded in the public Finance management information system. The information about budget revenues and expenditures accumulated in the database is used by a simulation model, which determines the values  $p_i, q_i$  for various variations of the state  $con_i = \frac{i}{m}$  of the automaton set by the researcher. These values are used to calculate the final probabilities  $\Phi_i^L, \Phi_i^H$  based on the obtained analytical expressions. The risk measure  $Con_i$  of the proposed solution is the probability values of the deficit  $q_i$  and surplus  $p_i$  determined using a simulation model. Figure 2 shows the use of model synthesis <stochastic automaton> and <simulation model> to determine the consequences of decisions made regarding the share  $con_i = \frac{i}{m}$  of tax revenue distribution between the budgets of a region and a municipality. According to Figure 2, the input of the simulation model from the database receives statistical data describing local budget revenues from local taxes, Federal taxes, as well as data on the amount of non-tax revenues and expenditures of the local budget. The probabilities of winning  $p_i$  and  $q_i$  losing machines, which are the output data of the simulation model, are determined based on computer experiments. An economic agent is in the process of making management decisions to conduct computer experiments by setting different values of standards for deductions from Federal taxes  $con_i \in CON$ .

#### 4.2. Discussion of Results

This article examines the concept of fiscal decentralization as a catalyst for economic development in the model of fiscal federalism, based on the analysis of works of foreign countries. It is noted that for many countries the problem of forming models of fiscal federalism is the joint ownership of Federal and territorial tax bases, as well as the need to solve the problem of determining the system of norms for the distribution of tax revenues between budgets of different levels of the budget system. Due to the fact that this problem cannot be investigated by purely theoretical methods, mathematical modeling becomes an unavoidable component in solving this problem. The need for a formal description of the behavior of the subject making a decision on the distribution of tax revenues, taking into account the qualitatively defined characteristics of the territories' ability for self-development, required the use of interdisciplinary approaches in the construction of a mathematical model. As part of the implementation of an interdisciplinary approach to the construction of a mathematical model, the article uses the synthesis of mathematical apparatus for stochastic automata operating in random environments, fuzzy algebra, and simulation. As a result of this synthesis, fuzzy automata are constructed that describe the behavior of the decision-making entity regarding the share of distribution of tax revenues to the budgets of subnational territories with a high and low level of self-organization. Fuzzy automata have the property of expediency of behavior, which is confirmed by its mathematically rigorous argumentation when proving the corresponding theorems. The obtained analytical expressions for the final probabilities of the automata staying in each of their States, interpreted as the values of the shares of deductions to the budget of the sub-region of revenues from a specific Federal tax, allow us to give a quantitatively justified assessment of the appropriateness of the decisions made. Due to the fact that the analytical expressions for the final probabilities include values  $p_i, q_i$ , that reflect respectively the probability of a budget surplus and deficit when choosing a standard  $con_i \in CON$ , the interaction of the automatic model with the simulation model is carried out to determine the quantitative values of these values. The simulation model forms a random environment formed by budget revenues and expenditures, in which the stochastic automaton is immersed. Statistical data on budget revenues and expenditures are obtained from the database of the budget management information system, in which has built-in mathematical models.

Thus, the decisions made  $con_i \in CON$  are evaluated using the values obtained at the output of the simulation model  $p_i, q_i$  and are used to determine the final probabilities  $\Phi_i^L, \Phi_i^H, i = \overline{1, m}$ . The results of experimental studies in determining the final probabilities  $\Phi_i^L, \Phi_i^H, i = \overline{1, m}$  are held for the tax on the income of individuals and shown in Table 2.

**Table 2.** Results of experimental studies in determining the final probabilities  $\Phi_i^L, \Phi_i^H, i = \overline{1, m}$ .

State of the Machine $con_i \in CON$	$con_1 = 0.25$	$con_2 = 0.5$	$con_3 = 0.75$	$con_4 = 1$
Final probabilities $\Phi_i^L$	0.353	0.312	0.2	0.135
Final probabilities $\Phi_i^H$	0.049	0.05	0.35	0.551

Final probabilities  $\Phi_i^L, \Phi_i^H, i = \overline{1, m}$  are interpreted as quantitative measures of the possibility of occurrence of events  $con_i \in CON$  in the presence of uncertainty in the situation of appropriate behavior of the decision-making subject. Experimental studies conducted with mathematical models provide the following recommendations. For sub-regions with a low level of self-development (Andevel class), in order to increase their level of budget security, it is more reasonable to use transfer payments, so the measure of expediency of assigning the values of the shares of deductions from the tax close to one is low: at  $con_4 = 1$  and  $con_3 = 0.75$  the final probabilities of the automaton take the values  $\Phi_4^L = 0.135$  and  $\Phi_3^L = 0.2$ . In this case, most of the tax deductions should go to the Federal budget. The need for transfer payments can be justified by "running" recommended solutions on a simulation model and assessing the likelihood of a budget deficit. For sub-regions of the Devel class that have a high level of self-development, the

task is to increase incentives for sub-regional authorities to increase their tax potential, and therefore, it is recommended to assign shares of tax deductions with a higher measure of expediency: At  $con_4 = 1$  and  $con_3 = 0.75$  the final probabilities of the automaton are respectively equal to  $\Phi_4^H = 0.551$ , and  $\Phi_3^H = 0.35$ . The simulation model allows you to predict the probability of budget deficits and surpluses under the established standards for deductions from tax.

In Table 3, the variables  $Con_i^H$  and  $Con_i^L$  indicate deductions from personal income tax for territories with a high (Devel class) and low (Andevel class) level of self-development, respectively.

**Table 3.** Estimates of probabilities of budget surpluses and deficits in sub-regions with different values  $con_i \in CON$  of deductions from personal income tax.

Subregion Devel	$Con_i^H$	0.25	0.5	0.75	1
	$p_i$	0.21	0.51	0.851	0.917
	$q_i$	0.79	0.49	0.149	0.083
Subregion Andevel	$Con_i^L$	0.25	0.5	0.75	1
	$p_i$	0.11	0.11	0.12	0.15
	$q_i$	0.89	0.89	0.88	0.85

In the future, it is planned to improve the constructed set of models, in which the data in Table 2 will be considered as a series of distributions of the value  $CON$  used for playing out the values of standards for deductions of Federal and regional tax revenues to the local budget using the method of statistical tests.

### 5. Conclusions

The research results allow us to come to the following conclusions. The hypothesis put forward in the article about the need for fiscal decentralization as the main path of self-development of the national economy of any country was confirmed by the analysis of scientific works of foreign scientists. The study of issues that arise in the formation of fiscal policy in various countries has demonstrated the existence of a global problem of creating conditions for sustainable economic development that ensure economic growth by increasing the tax base of sub-regions and thereby increasing their revenue autonomy. As confirmed in numerous scientific works, an effective way to increase the level of income independence of sub-regions is to reduce (if appropriate) the financial assistance provided to them and allow them to use part of the revenue from joint taxes collected in this territory. In this regard, there is a problem of developing tools for forming a system of norms for the distribution of tax revenues collected in a given territory between budgets of various hierarchical levels of administrative-territorial structure. Integral components of such tools are mathematical models that formally describe the behavior of the decision-making subject and allow us to give a quantitative justification for the chosen alternatives. The construction of these models required the use of an interdisciplinary approach. The proposed economic and mathematical tools in the form of a synthesis of automatic and simulation models are designed to support decision-making regarding the standards for the distribution of tax revenues between budgets of different levels of the hierarchy. The constructed automaton model describes the appropriate behavior of the decision-making economic agent. A mathematically rigorous proof of the adequacy of the automaton behavior is given. A conceptual scheme of interaction between automatic and simulation models functioning as part of an information system is presented. Based on the constructed mathematical models, experimental studies were carried out, which resulted in measures of expediency of establishing the shares of distribution of personal income tax between the local and Federal budgets of one of the regions of Russia. For the experiments, we used real data on budget revenues and expenditures of one of the municipalities of the Russian Federation. The theoretical significance of the results obtained in this work consists in the development of interdisciplinary approaches to the creation of mathematical models in the formation of

fiscal decentralization policy as a catalyst for economic development in the model of fiscal federalism.

The practical significance of the research lies in the possibility of using the constructed mathematical models to establish and coordinate the values of standards for the distribution of tax revenues between budgets of different levels of the hierarchy of the budget system.

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