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A Quantum Optical Approach to the effect of a Laser Mode on the Motion of Atomic Vapor by Varying the Field Coherence Angles

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Authors' contributions

This work was carried out in collaboration between both authors. Author AMA designed the study, performed the methodology, investigation, and wrote the first draft of the manuscript, supervision. Author AZE managed the analyses of the study, investigation, managed the literature searches. Both authors read and approved the final manuscript.

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ABSTRACT

We follow theoretically the motion of the sodium atoms in vapor state under the influence of a laser mode in (1 + 1) D, which is achieved by introducing different optical filters. In the Dirac interaction representation, the equations of motion are represented via the Bloch form together with the Pauli operators to find the elements of the density matrix of the system. The emergence of the principle of coherence in varying the angles of the laser mode permits the evaluation of the average force affecting the atoms' acceleration or deceleration, and hence the corresponding velocities and temperatures are investigated. The atomic vapor is introduced in a region occupied by a heat bath presented by the laser field, such that the state of the atomic vapor is unstable inside the system due to the loss or gain of its kinetic energy to or from the laser field. This instability is studied by

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finding the eigenvalues of the system's entropy. Resorting to the assumption of Botin, Kazantsev, and Pusep, who issued in the presence of the weak and strong spontaneous emission, a coupling between the mean numbers of photons in terms of time, which allows the evaluation of the rate of entropy production of the system under study. No singularities are found throughout the process of equations solving and other calculations. Resorting to symbolic software, a set of figures illustrating the nonlinear behavior in the dynamics of the problem is present. In this paper, we introduce a theoretical study of the effect of two-counter propagation traveling plane waves on the motion of the sodium atoms in the vapor state by varying the coherence angles to investigate the atomic behavior. Good agreements are found with previous studies.

Keywords: Laser pressure on atomic vapor; dirac representation; Coherent states; spontaneous emission; irreversible statistical mechanics; standing waves.

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1. INTRODUCTION

The prediction of the mechanical effects of laser light on the neutral atom's dates to Ashkin [1] and Kazantsev [2]. More specifically, in recent decades atomic beams have been laser-cooled and trapped, obtaining a narrowing of atomic lines and slowing down of atomic velocity. In the case of a plane monochromatic laser – traveling wave in the same direction as a two-level atomic beam, Kazantsev [2], Pusep [3] solved the equation of motion for the atomic density matrix with a semi classical Hamiltonian, and the field wave vector is taken in complex form that the resulting force lies in the complex plane, its real part takes the direction of the atomic beam and accelerates them, it was termed by the spontaneous radiation pressure force following Ashkin [1]. The imaginary part or the transverse component is the gradient or the dipole force which is in direct proportionality with the field intensity. Interaction representation has a long history in the study of atom dynamics in laser waves; the Dirac representation of a two-level atom and a nearly resonant light field had received a theoretical tackling in [4].

A work conducted by Letokhov and Minogin was based on the semi classical approach, such as in [5], who presented a quantum treatment based on the Schrödinger perception of the motion of atoms in a resonant light field. The interaction Hamiltonian is taken in the dipole and rotating wave approximations. Coherent states were first studied by Schrödinger in 1926, and were rediscovered by Klauder, Glauber, and Sudarshan at the beginning of the 1960's, they described the specific role of coherent states played in quantum radiation fields [6].

In reference [7], the acceleration of atoms by laser using the Landau-Lifshitz (LL) equation was discussed. Since laser cooling decreases the temperature of a sample of atoms, there is less disorder and therefore less entropy. This seems to conflict with the second law of thermodynamics, which requires the entropy of a closed system to always increase with time. The explanation lies in the consideration of the fact that in laser cooling the atoms does not form a closed system. Instead, there is always a flow of laser light with low entropy into the system and fluorescence with high entropy out of it. The decrease of entropy of the atoms is accompanied by a much larger increase in entropy of the light field. Entropy considerations for a laser beam are far from trivial, but recently it has been shown that the entropy lost by the atoms is many orders of magnitude smaller than the entropy gained by the light field [8].

The concept of entropy plays an important role in our understanding of complex physical systems. The study of the entropy in quantum systems was begun by von Neumann in 1932. The quantum entropy for a density operator was defined by von Neumann about 20 years before the Shannon entropy appeared. The quantum dynamical entropy (QDE) was studied in [9,10]. In 1976, Pusep has investigated the quantum features of the acceleration of an atom in the homogenous field of a traveling monochromatic wave. The atoms are accelerated as a result of absorption of photons of the traveling wave and of the spontaneous emission of a spherical wave. The improvement of the methodology utilized by Kazantsev [11] consists of the replacement of the classical description of the translational motion of the atom by a consistent quantum mechanical description. Indeed, upon absorbing a photon from the light flux and spontaneously emitting a spherical wave, the atoms acquire the momentum hk in the direction of propagation of the wave within a cycle of durationγ^{-1}. Despite that the spherical wave corresponds to the classical limit, from the quantum standpoint each elementary event of photon emission bears away a momentum equals in magnitude to hk in an arbitrary direction.

When the photon number n is characterized by a Poisson distribution, Botin and Kazanstev [2] and just after them Pusep [3] suggested that there is a linear relationship between the mean number of scattered photons and the time of laser-atom interaction in the form $\bar{n} = \gamma\, t/2$ if the spontaneous emission is strong, while$\bar{n} = 2\, G\, \gamma\, t$ if the spontaneous emission is weak, where γ^{-1} is the life time of the atomic levels. Since for $\bar{n} >> 1$ the distribution of $\bar{n}$ is well Gaussian approximated, while the distribution to which emission of photons gives rise in the case $n >> 1$ is also Gaussian. In 2016, Andrede et-al [12] studied the entropy of a quantized field in interaction with a two- level atom in a pure state when the field is initially in a mixture of two number states.

In this paper, a theoretical study discusses the effects on a neutral atomic vapor by a plane of monochromatic laser traveling wave in different coherent angles. The pressure force, the velocity and temperature of atoms were estimated. The standing laser wave [13] can be decomposed into a pair of oppositely traveling waves. In perturbation theory we can assume that these two acts on the atoms independently, and the resulting force is hence the direct superposition of two waves by combining up the two forces together to obtain a standing wave.

2. THE PHYSICAL PROBLEM

We consider a plane traveling wave, a single laser mode, of frequency ω propagating in the z-direction with wave vector ($\vec{K} = \pm k\vec{e}_z$), while a beam of vapor of Na atoms moves in the ($+ve$) z-direction. The role of the coherent state is played by introducing suitable optical filters inducing different angles ϕ to affect the state of motion of atoms making either the acceleration or deceleration acts, this is followed by a study which discusses the statistical nature of the quantum system via entropy and entropy production. The two-level model is employed to represent our problem. An atom with only two energy eigenvalues, is described as a two-dimensional state space spanning between the two energy eigenstates $|2\rangle$ and $|1\rangle$. The two states constitute a complete orthonormal system [14]. The corresponding energy eigenvalues are $E_1$and E_2. The energy levels of the atom are in a coordinate frame rotating with frequencyω_0.We use a type of laser withfrequencyωapproaching to the transition frequency ω_0 of sodium atoms. $\omega \sim \omega_0, \omega_0 = (\omega_1 - \omega_2)$; The atomic levels $|2\rangle$ and $|1\rangle$ are coupled by the light induced transitions, are separated in coordinate systems by a difference $\hbar\Delta\,, \Delta = \omega_0 - \omega$. The real value of the density matrix formalism for atom-light interactions is its ability to deal with open systems. The reason is that; the closed system of atom plus laser light that can be described by Schrödinger wave functions and is thus in a pure state, undergoes evolution to a ‘‘mixed’’ state by virtue of the spontaneous emission [8].

3. THE TACKLING APPROACH VIA DIRAC INTERACTION REPRESENTATION

The interaction Hamiltonian is taken in the dipole and rotating wave approximations in the framework of Dirac interaction representation [15] which takes the form;

$$\hat{H}_I = i\hbar\, g\{\hat{\pi}^\dagger \hat{a} e^{i\theta} - \hat{a}^\dagger \hat{\pi} e^{-i\theta}\}, \quad (1)$$

where $\hbar$ is the Planck constant, g is the Rabi frequency, the interaction between the laser and atoms is discussed depending on the distance and time for atoms such that;

$$\theta = \Delta t + \vec{K}.\vec{R} = \Delta t - kz, and\ \ \Delta = \omega_0 - \omega > 0,$$

here ω_0 is for atoms and while ω is for the field, $\vec{R} = z\underline{e}_z$is the atom position vector.

The operators $\hat{a}^\dagger$ and $\hat{a}$ are respectively the creation and destruction boson operators for the harmonic oscillator. We present the equation of motion of the matrix elements of the density matrix operator as$\hat{\rho}_I(t)$. The interaction of the two-level atom with the quantized electric field of an electromagnetic wave is defined by the Bloch equations [5,11]

$$\frac{d\rho_{ij}}{dt} = -\frac{i}{\hbar}\langle i|[\hat{H}_I, \rho_I]|j\rangle - \Gamma_{ij}(\rho_{ij} - \rho_{ij}^0), i, j = 1,2. \quad (2)$$

Where:

- $\Gamma_{11} = \Gamma_{22} = 2\gamma,$, are the spontaneous decay rates for the two metastable levels.

- $\Gamma_{12} = \Gamma_{21} = \gamma$, are the rates of relaxation of the off- diagonal matrix element.
- At t=0, we assume that the initial populations of the lower and upper states are $\rho_{11}^0 = 1, and\ \rho_{22}^0 = \rho_{12}^0 = \rho_{21}^0 = 0.$

The atom levels $|1\rangle\ and\ |2\rangle$ are coupled with the light induced transitions, are separated in coordinate systems with a difference $\hbar\,\Delta\,, \Delta = \omega_0 - \omega$.

Following [12] from (1) and (2), using the initial conditions for i = j = 1; the equations of motion are respectively (note that:$\bullet \equiv \frac{d}{dt}$)

$$\dot{\rho}_{11} = -2\gamma - 2\gamma\rho_{11} + g\{\hat{a}^{\dagger}\rho_{21}e^{-i\theta} + \rho_{12}\hat{a}e^{i\theta}\}, \quad (3.a)$$

similarly, for j = i = 2;

$$\dot{\rho}_{22} = -2\gamma\rho_{22} + g\{\hat{a}\rho_{12}e^{i\theta} + \rho_{21}\hat{a}^{\dagger}e^{-i\theta}\}, \quad (3.b)$$

for j = 2, i = 1;

$$\dot{\rho}_{12} = -\gamma\rho_{12} + g\{-\hat{a}^{\dagger}\rho_{22} + \rho_{11}\hat{a}^{\dagger}\}e^{-i\theta}, \quad (3.c)$$

for j = 1, i = 2;

$$\dot{\rho}_{21} = -\gamma\rho_{21} + g\{-\rho_{22}\hat{a} + \hat{a}\rho_{11}\}e^{i\theta}. \quad (3.d)$$

The density matrix satisfies the conditions for mixed states:

$$Tr\rho_I = 1, \rho_{12} = \rho_{21}^{\dagger}, \rho_I^2 \neq \rho_I.$$

Resorting to the coherent state $|\alpha\rangle$ of the laser mode, to obtain the value of density matrix elements according to which it is defined as an eigenstate of the amplitude operator, one of the annihilation operators $\{\hat{a}\}$, with eigenvalues $\{\alpha\}$ [16]. The operators $\{\hat{a}\}$ are non-Hermitian, and the phase angle φ describes the wave aspect of the coherent state $\alpha = |\alpha|e^{i\phi} \in C$, whereas α is a complex number, which corresponds to the complex wave amplitude in classical optics. Thus, the coherent states are wave-like states of the electromagnetic oscillators. As is well-known two beams of light are coherent when the phase difference between their waves is constant. So, in direct way the coherence angles ϕ are the phase of the electromagnetic field (laser) which affects the atoms. Although the coherent states are not orthogonal, it is possible to expand them in terms of a complete set of states. The completeness relation for the coherent states [17] and the following properties hold true:

$$\frac{1}{\pi}\int d^2\alpha|\alpha\rangle\langle\alpha| = 1 \quad , \quad \bar{\dot{\rho}}_{ij} = \langle\alpha|\dot{\rho}_{ij}|\alpha\rangle, \hat{a}|\alpha\rangle = \alpha|\alpha\rangle, \langle\alpha|\hat{a}^{\dagger} = \langle\alpha|\alpha^*, \langle\alpha|\alpha\rangle = 1. \quad (*)$$

So that applying (*) to both sides of equations (3) to get

$$\bar{\dot{\rho}}_{11} = 2\gamma - 2\gamma\overline{\rho}_{11} - g\{\alpha^*\overline{\rho}_{21}e^{-i\theta} + \alpha\overline{\rho}_{12}e^{i\theta}\}, \quad (4.a)$$

$$\bar{\dot{\rho}}_{22} = -2\gamma\overline{\rho}_{22} + g\{\alpha^*\overline{\rho}_{21}e^{-i\theta} + \alpha\overline{\rho}_{12}e^{i\theta}\}, \quad (4.b)$$

$$\bar{\dot{\rho}}_{12} = -\gamma\overline{\rho}_{12} + g\alpha^*\{\overline{\rho}_{22} - \overline{\rho}_{11}\}e^{-i\theta}, \quad (4.c)$$

$$\bar{\dot{\rho}}_{21} = -\gamma\overline{\rho}_{21} + g\alpha\{-\overline{\rho}_{22} + \overline{\rho}_{11}\}e^{i\theta} \quad (4.d)$$

4. PAULI OPERATORS [10], [11]

The Pauli matrices are a set of three 2 × 2 complex matrices which are Hermitian and unitary [17].

$$\hat{\sigma}_1 = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix} = \begin{bmatrix}0 & 1\\0 & 0\end{bmatrix} + \begin{bmatrix}0 & 0\\1 & 0\end{bmatrix} = \hat{\rho}_{12} + \hat{\rho}_{21}, \quad (5.a)$$

$$\hat{\sigma}_2 = \begin{bmatrix}0 & -i\\i & 0\end{bmatrix} = \begin{bmatrix}0 & 0\\i & 0\end{bmatrix} + \begin{bmatrix}0 & -i\\0 & 0\end{bmatrix} = i\begin{bmatrix}0 & 0\\1 & 0\end{bmatrix} - i\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix} = i(-\hat{\rho}_{12} + \hat{\rho}_{21}), \quad (5.b)$$

$$\hat{\sigma}_3 = \begin{bmatrix}1 & 0\\0 & -1\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 0\end{bmatrix} - \begin{bmatrix}0 & 0\\0 & 1\end{bmatrix} = \hat{\rho}_{11} - \hat{\rho}_{22}, \quad (5.c)$$

and

$$\hat{\rho}_{11} + \hat{\rho}_{22} = \hat{1}. \quad (5.d)$$

The determinants and traces of the Pauli matrices are:

$$det\,\sigma_i = -1, \quad Tr\sigma_i = 0. \quad (6)$$

The Pauli vector is defined by

$$\underset{\sim}{\sigma} = \sigma_x\hat{x} + \sigma_y\hat{y} + \sigma_z\hat{z}. \quad (7)$$

Then introducing coherent state $|\alpha\rangle$ in equations (5.1), (5.2), (5.3) and (5.4) leads to;

$$\bar{\sigma}_x = \bar{\sigma}_1 = (\bar{\rho}_{12} + \bar{\rho}_{21}, \bar{\sigma}_y = \bar{\sigma}_2 = i(\bar{\rho}_{21} - \bar{\rho}_{12}), \bar{\sigma}_z = \bar{\sigma}_3 = (\bar{\rho}_{11} - \bar{\rho}_{22}), 1 = (\bar{\rho}_{11} + \bar{\rho}_{22}). \quad (8)$$

Where $\hat{\sigma}_x$ and $\hat{\sigma}_y$ are Hermitian operators.

Differentiating (8) with respect to time and expressing in coherent states to obtain:

$$\bar{\dot{\sigma}}_x = (\bar{\dot{\rho}}_{12} + \bar{\dot{\rho}}_{21}) \ , \ \bar{\dot{\sigma}}_y = i(\bar{\dot{\rho}}_{21} - \bar{\dot{\rho}}_{12}) \ , \ \bar{\dot{\sigma}}_z = (\bar{\dot{\rho}}_{11} - \bar{\dot{\rho}}_{22}), 0 = (\bar{\dot{\rho}}_{11} + \bar{\dot{\rho}}_{22}). \quad (9)$$

Using equations (4) we have:

$$\dot{\bar{\sigma}}_x = -\gamma\sigma_x + 2g|\alpha|\cos(\theta+\phi)\bar{\sigma}_z, \quad (10.a)$$

$$\dot{\bar{\sigma}}_y = -\gamma\sigma_y - 2g|\alpha|Sin(\theta+\phi)\bar{\sigma}_z, \quad (10.b)$$

$$\dot{\bar{\sigma}}_z = -2\gamma - \gamma\bar{\sigma}_z - 2g|\alpha|\cos(\theta+\phi)\bar{\sigma}_x - 2g|\alpha|\sin(\theta+\phi)\bar{\sigma}_y. \quad (10.c)$$

The general compact form of those equations in (8) is

$$\dot{\bar{\sigma}} = M\bar{\sigma} + Y. \quad (10.d)$$

Where their matrix representations are respectively:

$$\bar{\sigma} = \begin{bmatrix}\bar{\sigma}_x\\ \bar{\sigma}_y\\ \bar{\sigma}_z\end{bmatrix}, Y = \begin{bmatrix}0\\0\\2\gamma\end{bmatrix}, M = \begin{bmatrix}-\gamma & 0 & 2gi|\alpha|\sin(\theta+\varphi)\\ 0 & \gamma & -2gi|\alpha|\cos(\theta+\varphi)\\ -2g|\alpha|\cos(\theta+\varphi) & -2g|\alpha|\sin(\theta+\varphi) & -2\gamma\end{bmatrix}.$$

The steady state solution is taken into consideration such that equation (10.d) will be:

$\dot{\bar{\sigma}} = 0$, this implies $\bar{\sigma} = -M^{-1}Y$; $|M| = -2\gamma^3 - 4g^2\bar{n}\gamma \neq 0$, with the inverted matrix

$$M^{-1} = \frac{1}{|M|}\begin{bmatrix}2\gamma^2 + 4g^2\bar{n}\sin^2(\theta+\varphi) & 4g^2\bar{n}\cos(\theta+\varphi)\sin(\theta+\varphi) & 2g\gamma|\alpha|\cos(\theta+\varphi)\\ 4ig\bar{n}\cos^2(\theta+\varphi) & 2\gamma^2 + 4g^2\bar{n}\cos^2(\theta+\varphi) & -2g\gamma|\alpha|\sin(\theta+\varphi)\\ -2g\gamma|\alpha|\cos(\theta+\varphi) & 2g\gamma|\alpha|\sin(\theta+\varphi) & \gamma^2\end{bmatrix}.$$

Hence Pauli operators in terms of the coherent states are:

$$\bar{\sigma}_x = \frac{-1}{|M|}4g\gamma^2|\alpha|\cos(\theta+\phi) = \bar{\rho}_{12} + \bar{\rho}_{21}. \quad (11.a)$$

$$\bar{\sigma}_y = \frac{-1}{|M|}(-4g\gamma^2|\alpha|\sin(\theta+\phi)) = i(\bar{\rho}_{12} - \bar{\rho}_{21}). \quad (11.b)$$

$$\bar{\sigma}_z = \frac{-1}{|M|}(2\gamma^3) = \bar{\rho}_{11} - \bar{\rho}_{22}, \quad (11.c)$$

provided that;

$$1 = \bar{\rho}_{11} + \bar{\rho}_{22}. \quad (11.d)$$

Two cases characterize the scope of this study:

The first case: the traveling wave is in the opposite direction to the atoms with detuning $\Delta > 0$.

The second case: the traveling wave is in the same direction of sodium atoms with detuning $\Delta > 0$.

Indeed; the density matrix elements under the effect of the coherent states are respectively

$$\bar{\rho}_{11} = \frac{1+0.5G\bar{n}}{1+G\bar{n}}, \bar{\rho}_{22} = \frac{0.5G\bar{n}}{1+G\bar{n}}, \bar{\rho}_{12} = \frac{g\alpha^* e^{-i\theta}}{\gamma(1+G\bar{n})}, \bar{\rho}_{21} = \frac{g\alpha e^{i\theta}}{\gamma(1+G\bar{n})}, \quad (12)$$

where $G = 2\left(\frac{g}{\gamma}\right)^2$ is the saturation parameter.

5. THE INVESTIGATION OF THE PROBLEM IN TERMS OF THE CONTINUOUS PHOTON NUMBERS

Recalling that, spontaneous emission causes the state of the system to evolve from a pure state into a mixed state and so the density matrix is needed to describe it. Spontaneous emission is an essential ingredient for the dissipative nature of the optical forces [18]. Thus, the stimulated transitions in the field of a running wave do not contribute to the mean force of the light pressure. We shall continue our investigations in two limiting cases of rapid and slow spontaneous emission.

5.1 The First Case

In order to investigate the effects resulting from the laser field on the pressure force, velocity and temperature of the atoms after interaction we proceed as follows:

The wave is traveling in the $(-ve)$ z-direction $(\vec{K}=-k\vec{e}_z)$ with $\theta_1=\Delta t-kz;\Delta>0$.

Using the completeness relation in (*); the density matrix elements therefore are:

$$\hat{\rho}_{11}=\frac{1+0.5G\hat{a}^{\dagger}\hat{a}}{1+G\bar{n}},\ \hat{\rho}_{22}=\frac{0.5G\hat{a}^{\dagger}\hat{a}}{1+G\bar{n}},\ \hat{\rho}_{12}=\frac{g\hat{a}^{\dagger}e^{-i\theta_1}}{\gamma(1+G\bar{n})},\ \hat{\rho}_{21}=\frac{g\hat{a}e^{i\theta_1}}{\gamma(1+G\bar{n})}. \quad (13)$$

Under the effect of an electromagnetic wave (laser), the force acting on a two-level atom in a resonance light field can be estimated as follows: in the field of a strong running wave, the atom absorbs a photon from a light beam and acquires the momentum $\hbar k$ of photon [19].

The Hamiltonian in matrix representation is

$$\hat{H}_I=\begin{bmatrix}0 & i\hbar g\hat{a}e^{i\theta_1}\\ -i\hbar g\hat{a}^{\dagger}e^{-i\theta_1} & 0\end{bmatrix}, \theta_1=\Delta t-kz.$$

Accordingly, the expression of the optical pressure force acting on atoms is

$$F=-\left\langle\frac{\partial H}{\partial z}\right\rangle=-Tr\{\rho_I\frac{\partial H}{\partial z}\}=-\frac{1}{4}\hbar kG\gamma\{(1-i)+\hat{a}^2e^{2i\theta_1}-\hat{a}^{\dagger 2}e^{-2i\theta_1}. \quad (14)$$

The value of the force with respect to the coherent state $|\alpha\rangle$ while using the relation (**) and the expansion of the exponential function is:

$$\bar{F}=\langle\alpha|F|\alpha\rangle=-\hbar k\gamma\, W_1;\, W_1=\frac{G\bar{n}}{1+G\bar{n}}\cos 2\,(\theta_1+\varphi). \quad (15.a)$$

The force is acting along the longitudinal direction and has real values, $\bar{n}$ is the mean number of photons.

We can divide both sides of equation (14) by $\hbar k\gamma$ to get its dimensionless form;

$$f=-W_1, \quad (15.b)$$

which causes a decelerating or an accelerating effect on the atoms along the negative z-direction. As is well-known, the probability of emission of a photon in a given direction is determined by the intensity of the spherical wave emitted by the quantum nature of the emission leads to fluctuation of the light-pressure force about the mean value of W_1.

We shall begin by integrating the force in (15.a). The relative velocity difference is expressed as

$$V=\frac{v(t)-v_0}{v_0}=-\frac{v_rG\bar{n}}{2\bar{\Delta}v_0(1+G\bar{n})}\{\sin 2\,(\theta_1+\phi)-\sin 2\,(\theta_0+\phi)\}. \quad (16.b)$$

In a one-dimensional space the kinetic energy $E_k=\frac{1}{2}Mv^2$ in the classical mechanics is equivalent to $E_k=\frac{1}{2}k_BT$ in thermodynamics, so that in the (-ve) z-direction the temperature of atoms will be

$$T(t)=\frac{M}{k_B}v^2=\frac{M}{k_B}\{v_0-\frac{v_rG\bar{n}}{2\bar{\Delta}(1+G\bar{n})}\{\sin 2\,(\bar{\theta}_1+\phi)-\sin 2\,(\theta_0+\phi)\}^2. \quad (17.a)$$

Such that the relative temperature difference reads:

$$T=\{1-\frac{v_rG\bar{n}}{2\bar{\Delta}v_0(1+G\bar{n})}\{\sin 2\,(\bar{\theta}_1+\phi)-\sin 2\,(\theta_0+\phi)\}^2. \quad (17.b)$$

5.2 The Second Case

The wave is traveling along the $(+ve)$ z-direction $(\vec{K}=k\vec{e}_z)$ with the new angle $\theta_2=\Delta t+kz$,

$\Delta>0$, and the density matrix elements are:

$$\hat{\rho}_{11}=\frac{1+0.5G\hat{a}^{\dagger}\hat{a}}{1+G\bar{n}},\hat{\rho}_{22}=\frac{0.5G\hat{a}^{\dagger}\hat{a}}{1+G\bar{n}},\ \hat{\rho}_{12}=\frac{g\hat{a}^{\dagger}e^{-i\theta_2}}{\gamma(1+G\bar{n})},\ \hat{\rho}_{21}=\frac{g\hat{a}e^{i\theta_2}}{\gamma(1+G\bar{n})}. \quad (18)$$

The Hamiltonian in matrix representation is

$$\hat{H}_I=\begin{bmatrix}0 & i\hbar g\hat{a}e^{i\theta_2}\\ -i\hbar g\hat{a}^{\dagger}e^{-i\theta_2} & 0\end{bmatrix}.$$

The pressure force in dimensionless form is:

$$f=W_2,\quad W_2=\frac{G\bar{n}}{1+G\bar{n}}\cos 2\,(\bar{\theta}_2+\phi),\ \text{where}\ \bar{\theta}_2=\bar{\Delta}\tau+2\pi Z. \quad (19)$$

It is acting along the positive z-direction, which causes decelerating or accelerating effects on the atoms for chosen values of the phase angles.

The relative velocity difference is:

$$V=\frac{v_rG\bar{n}}{2\bar{\Delta}v_0(1+G\bar{n})}\{\sin 2\,(\bar{\theta}_2+\phi)-\sin 2\,(\theta_0+\phi)\}. \quad (20)$$

Similarly, as in the first case, the corresponding relative temperature difference reads:

$$T = \{\frac{v_r G\bar{n}}{2\bar{\Delta} v_0 (1+G\bar{n})}\{sin\,2\,(\bar{\theta}_2 + \phi) - sin\,2\,(\theta_0 + \phi)\}^2. \quad (21)$$

6. THE INVESTIGATION OF THE PROBLEM IN TERMS OF TIME REPRESENTATION

The idea of coupling between the mean number of photons and time was introduced by Botin and Kasantsev [2] and Pusep [3]; they adopted that the distribution of the absorbed photons *n* is well approximated as Poissonian or Gaussian, according to whether the spontaneous emission is slow or rapid. In the first, the emitted mean photon numbers $\bar{n}$ arises as Gaussian, it is linearly proportional to time such as $\bar{n} = 0.5\ \tau;\ \tau = \gamma\ t$, it is also known as strong field. For rapid spontaneous emission *n* is Gaussian and the mean photon numbers $\bar{n}$ is Gaussian as well, where $\bar{n} = 2G\tau$, it is known also as weak field where $G \ll 1$. Here τ is the interaction time (dimensionless), while for slow spontaneous emission $G \gg 1$.

6.1 Slow Spontaneous Emission (Strong Field): [2,3]

6.1.1 First case

The wave is traveling in the (-ve) z-direction with $\theta_1 = \Delta t - kz$ and, $\Delta > 0$. The density matrix elements are**:**

$$\hat{\rho}_{11} = \frac{1 + 0.5\,G\,\hat{a}^{\dagger}\hat{a}}{1 + 0.5\,G\,\tau},\ \hat{\rho}_{22} = \frac{0.5\,G\,\hat{a}^{\dagger}\hat{a}}{1 + 0.5\,G\,\tau},$$
$$\hat{\rho}_{12} = \frac{g\,\hat{a}^{\dagger}\,e^{-i\theta_1}}{\gamma(1 + 0.5\,G\,\tau)},\ \hat{\rho}_{21} = \frac{g\,\hat{a}\,e^{i\theta_1}}{\gamma(1 + 0.5\,G\,\tau)}. \quad (22)$$

The force, according to the mean photon number in terms of a time representation under the strong field is;

$$f = -\frac{G\,\tau}{2 + G\,\tau}\cos 2(\bar{\theta}_1 + \phi)\,. \quad (23.a)$$

The corresponding atoms velocity;

$$V = -\frac{v_r\,G}{v_0}\int_{\tau_0}^{\tau}\frac{\tau}{(2 + G\,\tau)}\cos 2(\bar{\theta}_1 + \phi)\,d\tau\,. \quad (23.b)$$

Together with the atom’s temperature;

$$T = \{1 - \frac{v_r\,G}{v_0}\int_{\tau_0}^{\tau}\frac{\tau}{(2 + G\,\tau)}\cos 2(\bar{\theta}_1 + \phi)\,d\tau\}^2. \quad (23.c)$$

6.1.2 Second case

The wave is traveling in the (+ve) z-direction with $\bar{\theta}_2 = \bar{\Delta}\tau + 2\pi Z$ and $\Delta > 0$, the density matrix elements are**:**

$$\hat{\rho}_{11} = \frac{1 + 0.5\,G\,\hat{a}^{\dagger}\hat{a}}{1 + 0.5\,G\,\tau},\ \hat{\rho}_{22} = \frac{0.5\,G\,\hat{a}^{\dagger}\hat{a}}{1 + 0.5\,G\,\tau},$$
$$\hat{\rho}_{12} = \frac{g\,\hat{a}^{\dagger}\,e^{-i\theta_2}}{\gamma(1 + 0.5\,G\,\tau)},\ \hat{\rho}_{21} = \frac{g\,\hat{a}\,e^{i\theta_2}}{\gamma(1 + 0.5\,G\,\tau)}. \quad (24)$$

The force, according to the mean photon number in terms of a time representation with a strong field is;

$$f = \frac{G\,\tau}{2 + G\,\tau}\cos 2(\bar{\theta}_2 + \phi)\,. \quad (25.a)$$

The corresponding atoms velocity;

$$V = \frac{v_r\,G}{v_0}\int_{\tau_0}^{\tau}\frac{\tau}{(2 + G\,\tau)}\cos 2(\bar{\theta}_2 + \phi)\,d\tau\,. \quad (25.b)$$

Together with the atom’s temperature;

$$T = \{1 - \frac{v_r\,G}{v_0}\int_{\tau_0}^{\tau}\frac{\tau}{(2 + G\,\tau)}\cos 2(\bar{\theta}_2 + \phi)\,d\tau\}^2. \quad (25.c)$$

6.2 Rapid Spontaneous Emission (weak Field)

6.2.1 First case

The wave is traveling in the (- ve) z-direction with $\theta_1 = \Delta t - kz$ and, $\Delta > 0$ .The density matrix elements are**:**

$$\hat{\rho}_{11} = \frac{1 + 0.5\,G\,\hat{a}^{\dagger}\hat{a}}{1 + 2\,G^2\,\tau},\quad \hat{\rho}_{22} = \frac{0.5\,G\,\hat{a}^{\dagger}\hat{a}}{1 + 2\,G^2\,\tau},$$
$$\hat{\rho}_{12} = \frac{g\,\hat{a}^{\dagger}\,e^{-i\theta_1}}{\gamma(1 + 2\,G^2\,\tau)},\ \hat{\rho}_{21} = \frac{g\,\hat{a}\,e^{i\theta_1}}{\gamma(1 + 2\,G^2\,\tau)}. \quad (26)$$

The force in terms of the mean photons number in time representation under the weak field

$$f = \frac{2G^2\tau}{1+2G^2\tau} cos\, 2\,(\bar{\theta}_1 + \phi). \quad (27.a)$$

The corresponding velocity is;

$$V = \frac{2G^2 v_r}{v_0} \int_{\tau_0}^{\tau} \frac{\tau}{1+2G^2\tau} cos\, 2\,(\bar{\theta}_1 + \phi) d\tau, \quad (27.b)$$

together with the temperature

$$T = \{1 - \frac{2G^2 v_r}{v_0} \int_{\tau_0}^{\tau} \frac{\tau}{1+2G^2\tau} cos\, 2\,(\bar{\theta}_1 + \phi) d\tau\}^2. \quad (27.c)$$

6.2.2 Second case

The wave is traveling in the (+ve) z-direction with $\bar{\theta}_2 = \bar{\Delta}\tau + 2\pi Z$ and $\Delta > 0$ .The density matrix elements are**:**

$$\hat{\rho}_{11} = \frac{1 + 0.5\, G\, \hat{a}^{\dagger} \hat{a}}{1 + 2G^2 \tau} \;, \hat{\rho}_{22} = \frac{0.5\, G\, \hat{a}^{\dagger} \hat{a}}{1 + 2G^2 \tau} \;,$$

$$\hat{\rho}_{12} = \frac{g\, \hat{a}^{\dagger} e^{-i\theta_2}}{\gamma (1 + 2G^2 \tau)} \;, \hat{\rho}_{21} = \frac{g\, \hat{a}\, e^{i\theta_2}}{\gamma (1 + 2G^2 \tau)}. \quad (28)$$

The force in terms of the mean photons number in time representation under the weak field

$$f = \frac{2G^2 \tau}{1 + 2G^2 \tau} \cos 2(\bar{\theta}_2 + \phi)\,. \quad (29.a)$$

The corresponding velocity is;

$$V = \frac{2 v_r\, G^2}{v_0} \int_{\tau_0}^{\tau} \frac{\tau}{(1 + 2G^2 \tau)} \cos 2(\bar{\theta}_2 + \phi)\, d\tau \cdot \quad (29.b)$$

together with the temperature;

$$T = \{1 + \frac{2G^2 v_r}{v_0} \int_{\tau_0}^{\tau} \frac{\tau}{1+2G^2\tau} cos\, 2\,(\bar{\theta}_2 + \phi) d\tau. \quad (29.c)$$

7. THE INTERFERENCE (SUPER-POSITION) OF TWO OPPOSITELY DIRECTED LASER PLANE WAVES AFFECTING SODIUM VAPOR: VERIFICATION VIA STENHOLM TACKLING

The standing laser wave can be decomposed into a pair of oppositely traveling waves. We can apply the force in the (-ve - eq. (15.b)- and in (+ve - eq. (19)- directions from the previous sections and the usage of Stenholm suggestion "Adding between two different direction gives a standing wave" [13].

7.1 The Investigation of the Problem in Terms of the Continuous Photon Numbers

The acting force on atoms in dimensionless form;

$$f = \frac{G\bar{n}}{1+G\bar{n}} (- cos\, 2\,(\theta_1 + \phi) + cos\, 2\,(\theta_2 + \phi)), \quad (30.a)$$

where $\theta_1 = (\bar{\Delta}\tau - 2\pi z), and\ \theta_2 = (\bar{\Delta}\tau + 2\pi z)$.

The corresponding velocity is;

$$V = \frac{v_r G\bar{n}}{2\bar{\Delta}(1+G\bar{n})} \{- sin\, 2\,(\theta_1 + \phi) + sin\, 2\,(\theta_2 + \phi)\}. \quad (30.b)$$

together with the dimensionless temperature T which is proportional to V^2

$$T + C = T_1 + T_2. \quad (30.c)$$

7.2 The Investigation of the Problem in Terms of Time

7.2.1 Slow spontaneous emission

The emitted mean photon numbers $\bar{n}$ gives rise also to be Gaussian, it is linearly proportional to time such as $\bar{n} = 0.5\ \tau;\ \tau = \gamma\ t$, it is known also as strong field.

Accordingly, the total force acting on atoms using eqs. (23.a) and (25.a) is;

$$f = -\frac{G\tau}{2+G\tau} \left(cos\, 2\,(\bar{\theta}_1 + \phi) - cos\, 2\,(\theta_2 + \phi) \right). \quad (31.a)$$

From eqs. (23.b) and (25.b), the resulting velocity is therefore;

$$V = -\frac{v_r G}{v_0} \left(\int_{\tau_0}^{\tau} \frac{\tau}{(2+G\tau)} cos\, 2\,(\bar{\theta}_1 + \phi)\, d\tau - \right.$$
$$\tau 0\tau\tau(2+G\tau)cos2(\theta 2+\phi)d\tau. \quad (31.b)$$

The dimensionless energy equation per unit mass is;

$V^2 + C = {v_1}^2 + {v_1}^2$, such that $T + C = T_1 + T_2$.

Using eqs (23.c) and (25.c) the total temperature will amount to;

$$T + C = \{1 - \frac{v_r G}{v_0} \int_{\tau_0}^{\tau} \frac{\tau}{(2 + G\tau)} \cos 2(\bar{\theta}_1 + \phi)\, d\tau\}^2$$
$$+ \{1 + \frac{v_r G}{v_0} \int_{\tau_0}^{\tau} \frac{\tau}{(2+G\tau)} \cos 2(\bar{\theta}_2 + \phi)\, d\tau\}^2. \quad (31.c)$$

7.2.2 Rapid spontaneous emission

Resorting to the relation between the mean photons number in the weak field $\bar{n} = 2G\tau : G \ll 1$.
Accordingly, we get the total force acting on atoms using eqs. (27.a) and (29.a);

$$f = -\frac{G\tau}{2+G\tau}\left(\cos 2(\bar{\theta}_1 + \phi) - \cos 2(\theta_2 + \phi)\right). \quad (31.a)$$

From eqs. (27.b) and (29.b), the resulting velocity is therefore;

$$V = -\frac{v_r}{v_0}\left(\int_{\tau_0}^{\tau} \frac{2G^2\tau}{1+2G^2\tau} \cos 2(\bar{\theta}_1 + \phi)\, d\tau - \int_{\tau_0}^{\tau} \frac{2G^2\tau}{1+2G^2\tau} \cos 2(\bar{\theta}_2 + \phi)\, d\tau\right). \quad (31.b)$$

The dimensionless energy equation per unit mass is;

$V^2 + C = {v_1}^2 + {v_1}^2$, such that $T + C = T_1 + T_2$.

Using eqs. (27.c) and (29.c) the total temperature will amount to

$$T + C = \{1 - \frac{v_r}{v_0} \int_{\tau_0}^{\tau} \frac{2G^2\tau}{1+2G^2\tau} \cos 2(\bar{\theta}_1 + \phi d\tau\}2+\{1+vrv0\tau0\tau2G2\tau1+2G2\tau cos2\theta2+\phi d\tau\}^2. \quad (31.c)$$

8. THE IRREVERSIBLE STATISTICAL DYNAMICS OF THE PROBLEM

8.1 Entropy of the System

The laser field in this study is suggested as a heat bath. Indeed, the most natural measure of the uncertainty of the quantum- mechanical state is the entropy. The quantum mechanical entropy due to Von-Neumann reads [20] is;

$$S_{\hat{\rho}} = -k_B Tr\{\hat{\rho} ln \hat{\rho}\}, \hat{\rho} = \hat{\rho}_{2x2}, \quad (32.a)$$

where $\hat{\rho}$ the density operator of the quantum mechanical system, k_B is the Boltzmann's constant. The Von-Neumann entropy of a pure state is equal to zero. For the quantum mechanical mixture, the Von-Neumann entropy is larger than zero.

By deriving the eigenvalues of density matrix **[21]**

$$S_{\hat{\rho}} = -k_B \sum_{l=1}^{d} \lambda_l \log \lambda_l = -k_B\{\lambda_1 \log \lambda_l + \lambda_2 \log \lambda_2\}, \quad (32.b)$$

to identify the eigenvalues of the density matrix from the eigenequation

$$\begin{vmatrix} \bar{\rho}_{11} - \lambda & \bar{\rho}_{12} \\ \bar{\rho}_{21} & \bar{\rho}_{22} - \lambda \end{vmatrix} = 0 \Rightarrow \lambda^2 - \lambda + \bar{\rho}_{11}\bar{\rho}_{22} - \bar{\rho}_{12}\bar{\rho}_{21} = 0.$$

Using eq. (12) we get:

$$\lambda_{1,2} = \frac{1}{2} \pm \frac{\sqrt{1-2G\bar{n}}}{2(1+G\bar{n})}. \quad (32.c)$$

According to **[22],** the stability of the system is described according to the eigenvalues in the form

$$\lambda_1 = \bar{\lambda}_1 = a + ib : a > 0 \;\; and \;\; b > 0.$$

So that the system behaves as a "unstable spiral " meaning that the atoms dissipate /acquire energy to/ from the laser field as an energy bath described by the variable mean photon numbers $\bar{n}$..

Entropy has the dimension of Boltzmann constant k_B; its dimensionless form will be;

$$S = \frac{S_{\hat{\rho}}}{k_B} = -\{\lambda_1 \log \lambda_l + \lambda_2 \log \lambda_2\}. \quad (32.d)$$

In the conventional expression of the field in terms of the photon numbers, ***S*** is independent of time; as a result, the entropy production could not be evaluated.

8.1.1 For slow spontaneous emission

Resorting to Kasantsev and Pusep who adopted the notion of strong and weak fields in terms of time, which permits to investigate the entropy production. The eigenvalues depend on time, therefore

The eigenvalues in the strong field are

$$\lambda_{1,2} = \frac{1}{2} \pm \frac{\sqrt{1-G\tau}}{(2+G\tau)}, \quad (33.a)$$

hence, the dimensionless entropy reads;

$S_{str} = -\{\lambda_1 \log \lambda_l + \lambda_2 \log \lambda_2\}, \lambda_{1,2} = \lambda_{1,2}(t)$.(33.b)

It follows that the entropy production will be;

$$\sigma_{st} = \frac{d}{d\tau} S_{st}, \tag{34.a}$$

$$\sigma_{st} = \frac{G\left[(-2+0.5G\tau)Log\left[0.5-\frac{\sqrt{1-G\tau}}{(2+G\tau)}\right]+(2-0.5G\tau)Log\left[0.5+\frac{\sqrt{1-G\tau}}{(2+G\tau)}\right]\right]}{\sqrt{1-G\tau}(2+G\tau)^2} \tag{34.b}$$

8.1.2 For rapid spontaneous emission

The eigenvalues in the weak field are;

$$\lambda_{1,2} = \frac{1}{2} \pm \frac{\sqrt{1-2G^2\tau}}{2(1+2G^2\tau)}, \tag{35.a}$$

hence, the dimensionless entropy in the strong field is;

$$S_w = -\{\lambda_1 \log \lambda_l + \lambda_2 \log \lambda_2\}.. \tag{35.b}$$

Here S_w is time dependent, as a result the entropy production $\dot{S}=\sigma$,, could be calculated.

$$\sigma_w = \frac{d}{d\tau} S_w, \tag{36.a}$$

$$\sigma_w = \frac{G^2\left[(-1.5+G^2\tau)Log\left[0.5-\frac{\sqrt{1-2G^2\tau}}{(2+4G^2\tau)}\right]+(1.5-G^2\tau)Log\left[0.5+\frac{\sqrt{1-2G^2\tau}}{(2+4G^2\tau)}\right]\right]}{\sqrt{1-2G^2\tau}(1+2G^2\tau)^2}. \tag{36.b}$$

9. THE RESULTS AND DISCUSSION

It is worth noting that throughout all the evaluations of the integrals in the study and consequently the corresponding illustrations no singularities arise. This will be seen by the help of "Mathematica" software. Besides, no approximations were introduced.

The angles of the interaction between the atoms and the waves are illustrated and show linear periodic oscillations of the force acting on the atom's behavior for several values of the coherence angles. The induced changes are in distance z and time t, see Figs. (1.a) and (1.b).

According to Table (1) the coherent and interaction angles are presented in some order, which reveals the physical degradation of acceleration Figs. (2.a), (2.b), the deceleration behavior of atoms in Figs (2.c), (2.d), and their corresponding relative temperature difference.

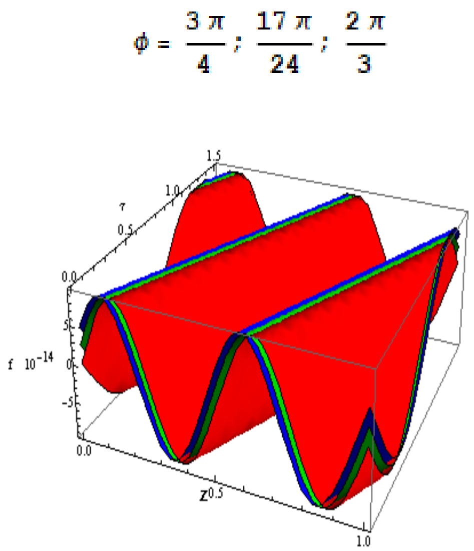


Fig. (1.a)

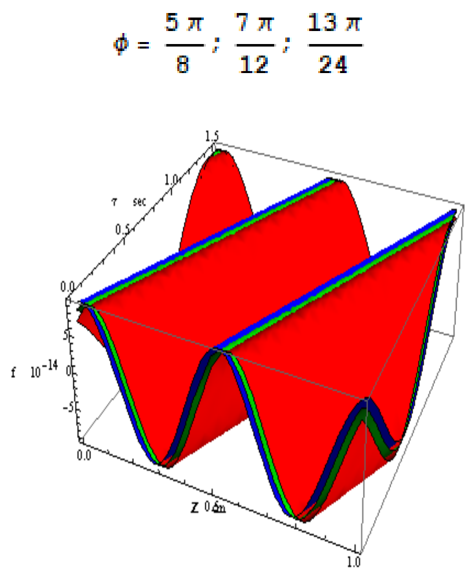


Fig. (1.b)

Figs. (1.a), (1.b) show the force acting on the atoms in time and space.

Table 1.

ϕ	θ	ϕ	θ
$\pi/6$	$\pi/3$	$\pi/3$	$5\pi/6$
$\pi/3$	$\pi/6$	$2\pi/3$	$\pi/3$
$\pi/2$	$\pi/6$	$5\pi/6$	$\pi/3$

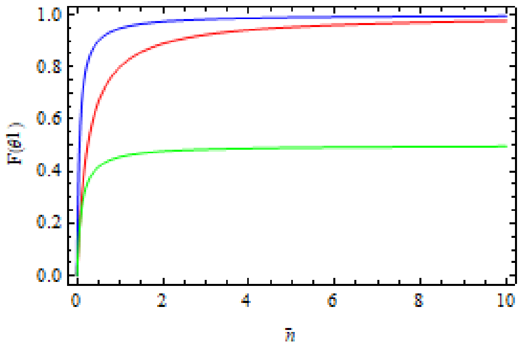


Fig. (2.a)

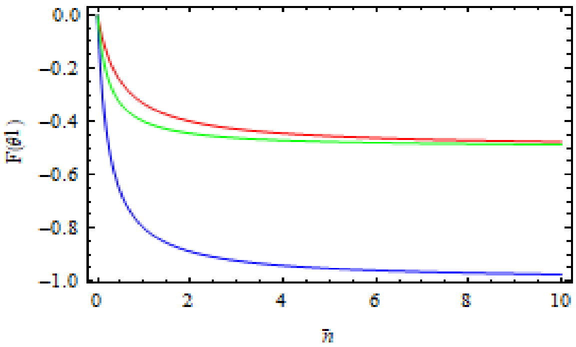


Fig. (2.c)

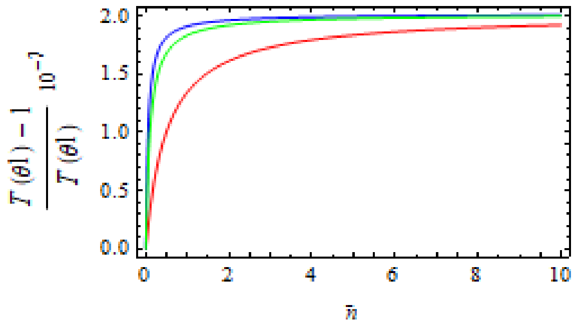


Fig. (2.b)

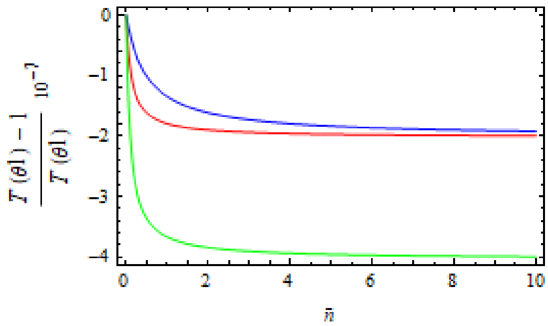


Fig. (2.d)

Figs. (2.a) and (2.b) illustrate the increasing in the force and temperature according to the coherent angles. Figs. (2.c) and (2.d) illustrate the decreasing in the force and temperature according to the coherent angles. These figures describe the deceleration or acceleration in the atom's behavior.

According to Table (2) the coherent and interaction angles are almost in opposite order, which illustrates both the acceleration in Figs. (3.a), (3.b), and deceleration in Figs. (3.c), (3.d) of the atoms, and their corresponding relative temperature difference.

The investigation of the problem, according to Kazantsev and Pusep in terms of time reveals, in the cases:

9.1 Slow Spontaneous Emission

The mean number of photons in the time representation is $\bar{n} = 0.5\tau$, where the field is strong, and the saturation coefficient has taken such that $G >> 1$. The proper choice of coherence angles also controls the behavior of the atoms, which are shown according to Tables and Figures.

In Table 3 For the first case the coherent and interaction angles are in several orders, which illustrates both the acceleration and deceleration behavior of the atoms respectively as shown in Figs. (4.a), (4.b) and Figs. (4.c), (4.d), and their corresponding relative temperature difference.

Table 2.

ϕ	θ	ϕ	θ
$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{\pi}{6}$	$\frac{\pi}{3}$
$\frac{2\pi}{3}$	$\frac{\pi}{3}$	$\frac{\pi}{3}$	$\frac{\pi}{6}$
$\frac{5\pi}{6}$	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{\pi}{6}$

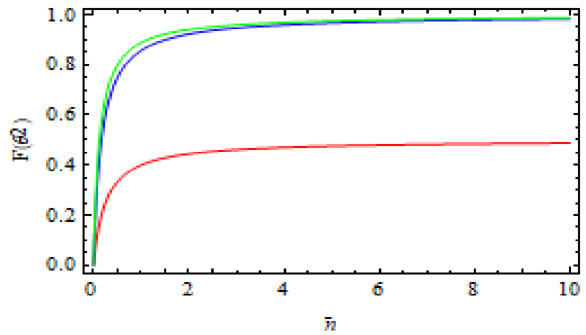


Fig. (3.a)

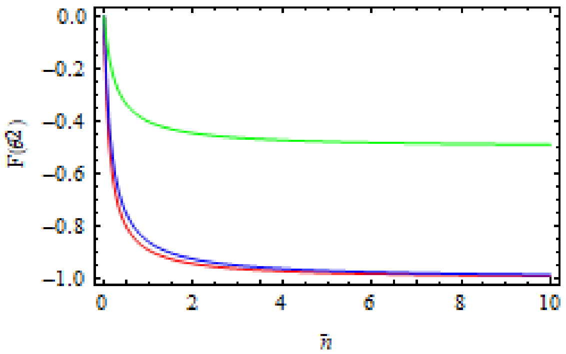


Fig. (3.c)

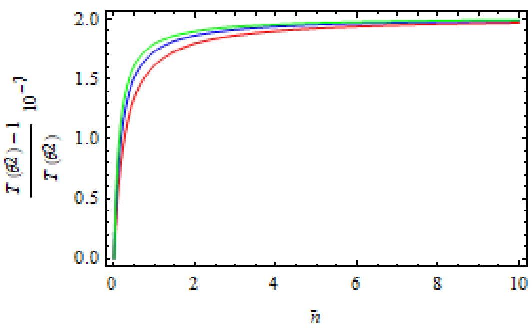


Fig. (3.b)

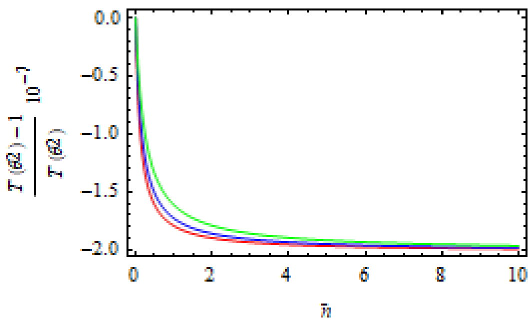


Fig. (3.d)

Figs. (3.a) and (3.b) illustrate the increasing in the force and temperature according to the coherent angles. Figs. (3.c) and (3.d) illustrate the decreasing in the force and temperature according to the coherent angles. These figures, which describe the deceleration or acceleration in the atom's behavior.

Table 3

ϕ	θ	ϕ	θ
$\pi/6$	$\pi/2$	$\pi/3$	$5\pi/6$
$\pi/2$	$\pi/6$	$\pi/2$	$2\pi/3$
$\pi/3$	$\pi/3$	$2\pi/3$	$\pi/2$
$5\pi/6$	$5\pi/6$	$5\pi/6$	$\pi/3$

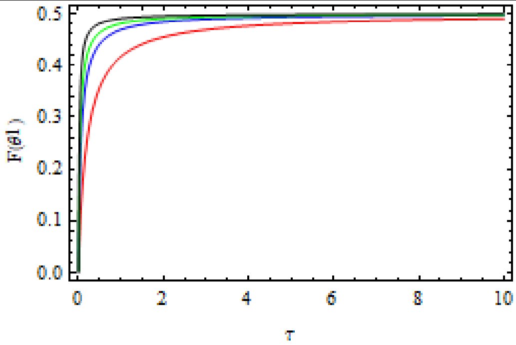


Fig. (4.a)

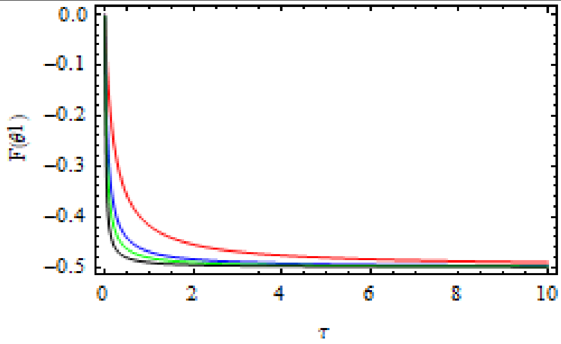


Fig. (4.c)

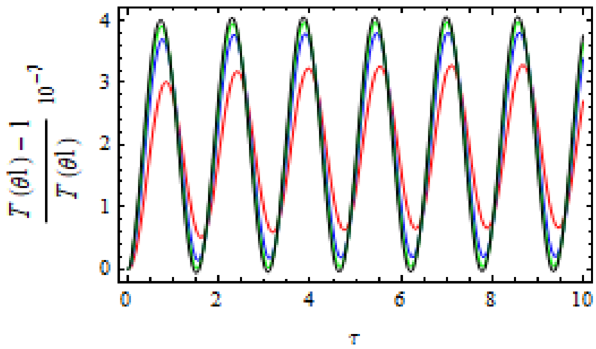


Fig. (4.b)

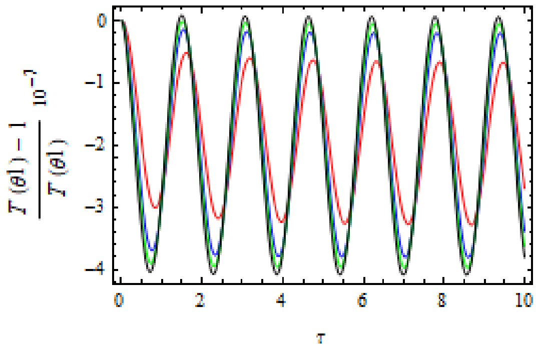


Fig. (4.d)

Figs. (4.a) and (4.b) illustrate the increasing in the force and temperature according to the coherent angles. Figs. (4.c) and (4.d) illustrate the decreasing in the force and temperature according to the coherent angles. These figures, which describe the deceleration or acceleration in the atom's behavior

In Table (4) according to the third case the coherent and interaction angles are preseted in different orders, which illustrates both the acceleration and deceleration behaviors of the atoms respectively shown in Figs. (5.a) , (5.b) and Figs. (5.c) , (5.d) and their corresponding relative temperature difference.

9.2 Rapid Spontaneous Emission

The mean number of photons was treated such that $\bar{n} = 2G\tau$, where the field is weak i.e. $G <<$ *1* . The coherence angles also control the behavior of the atoms, which are shown.

Table 5 is related to the first case, the coherent and interaction angles are presented in some order, which illustrates both the acceleration and deceleration behavior of the atoms respectively, are shown in Figs. (6.a) , (6.b) and Figs. (6.c) , (6.d) and their corresponding relative temperature difference.

Table 4

ϕ	θ	ϕ	θ
$\frac{\pi}{3}$	$\frac{5\pi}{6}$	$\frac{\pi}{6}$	$\frac{\pi}{2}$
$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{\pi}{2}$	$\frac{\pi}{6}$
$\frac{2\pi}{3}$	$\frac{\pi}{2}$	$\frac{\pi}{3}$	$\frac{\pi}{3}$
$\frac{5\pi}{6}$	$\frac{\pi}{3}$	$\frac{5\pi}{6}$	$\frac{5\pi}{6}$

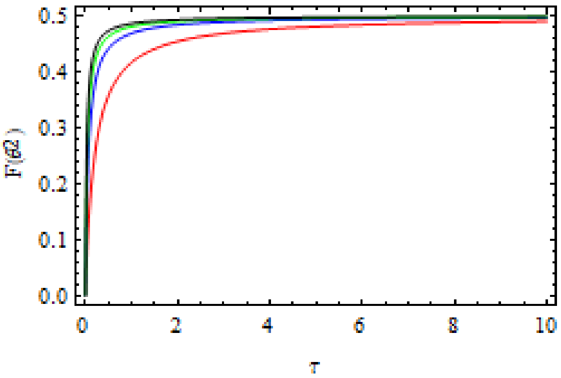


Fig. (5.a)

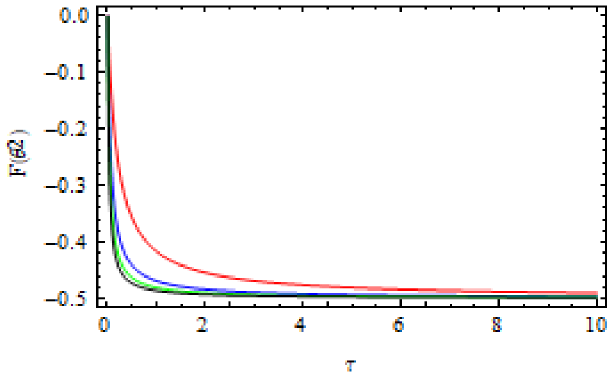


Fig. (5.c)

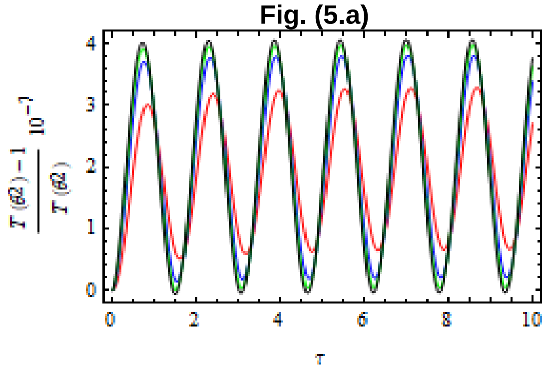


Fig. (5.b)

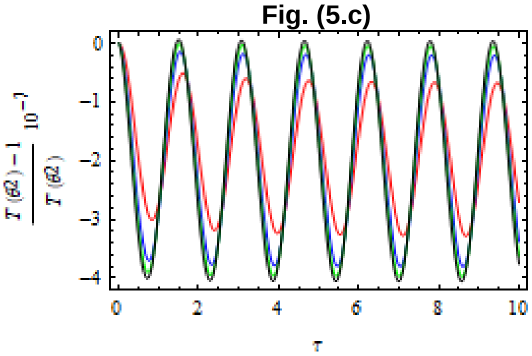


Fig. (5.d)

Figs. (5.a) and (5.b) illustrate the increasing in the force and temperature according to the coherent angles. Figs. (5.c) and (5.d) illustrate the decreasing in the force and temperature according to the coherent angles. These figures, which describe the deceleration or acceleration in the atom's behavior representing in the time

Table 6 illustrates the case, where the coherent angles and interaction angles are presented in opposite order, which show both the acceleration and deceleration behaviors of the atoms respectively, see in Figs. (7.a), (7.b) and Figs. (7.c), (7.d) and their corresponding relative temperature difference.

The studying of the state of equilibrium in the system is accomplished by evaluating the entropy in the classical representation of the field, Fig. (8) Explains the situation that the system behaves as an "unstable spiral". While in terms of time, comparing both slow and rapid spontaneous emission cases, regardless of the coherence angles; the entropy production behaves such that it is suddenly increases with increasing G at the beginning of the time interval, then drops severely to zero during the rest of the time interval approaching the state of equilibrium with an increase in entropy, where the energy is dissipated to the field, in agreement with the notion of open systems [9], As shown in Figs. (9.a) and (9.b).

Table 5.

ϕ	θ	ϕ	θ
$\frac{\pi}{6}$	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$
$\frac{\pi}{2}$	$\frac{5\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\frac{2\pi}{3}$	$\frac{2\pi}{3}$	$\frac{\pi}{2}$	$\frac{\pi}{3}$
$\frac{5\pi}{6}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{\pi}{6}$

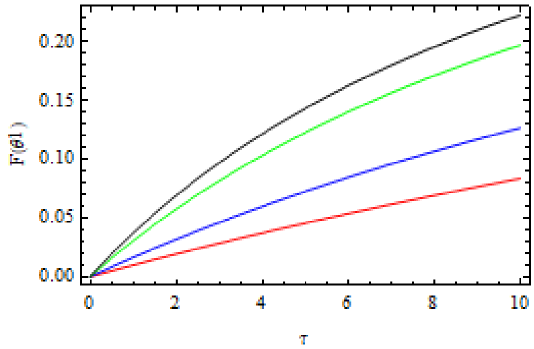


Fig. (6.a)

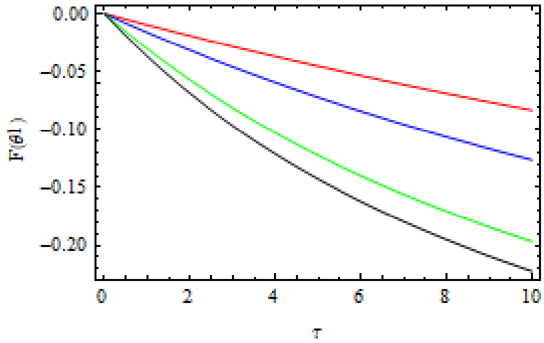


Fig. (6.c)

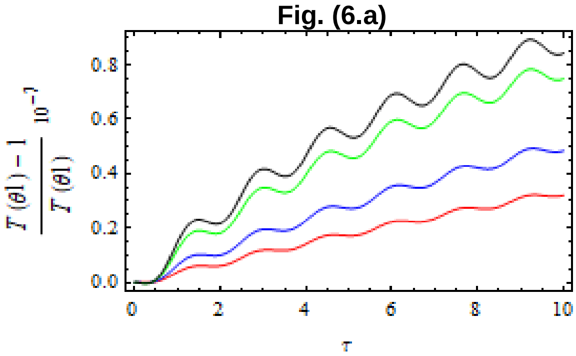


Fig. (6.b)

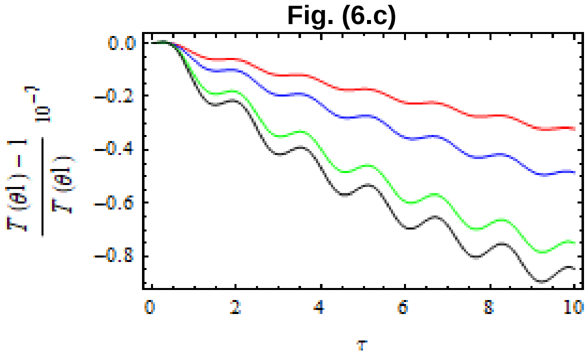


Fig. (6.d)

Figs. (6.a) and (6.b) illustrate the increasing in the force and temperature according to the coherent angles. Figs. (6.c) and (6.d) illustrate the decreasing in the force and temperature according to the coherent angles. These figures, which describe the deceleration or acceleration in the atom's behavior representing in the time

Table 6.

ϕ	θ	ϕ	θ
$\frac{\pi}{3}$	$\frac{5\pi}{6}$	$\frac{\pi}{6}$	$\frac{\pi}{2}$
$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{\pi}{3}$	$\frac{\pi}{3}$
$\frac{2\pi}{3}$	$\frac{\pi}{2}$	$\frac{\pi}{2}$	$\frac{\pi}{6}$
$\frac{5\pi}{6}$	$\frac{\pi}{3}$	$\frac{5\pi}{6}$	$\frac{5\pi}{6}$

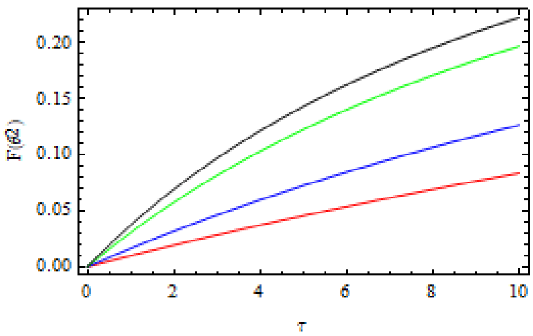


Fig. (7.a)

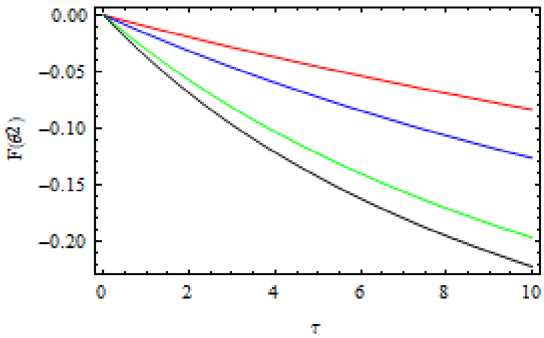


Fig. (7.c)

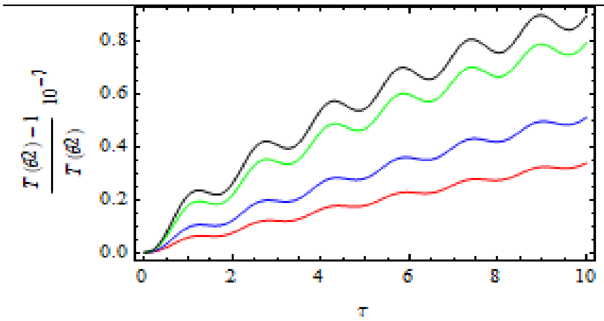


Fig. 7.b.

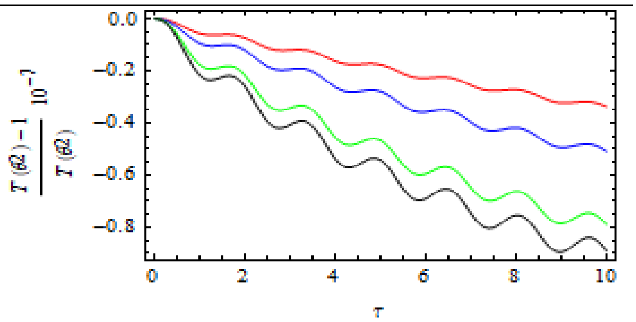


Fig. 7.d.

Figs. 7.a. and (7.b) illustrate the increasing in the force and temperature according to the coherent angles. Figs. (7.c) and (7.d) illustrate the decreasing in the force and temperature according to the coherent angles. These figures, which describe the deceleration or acceleration in the atom's behavior representing in the time

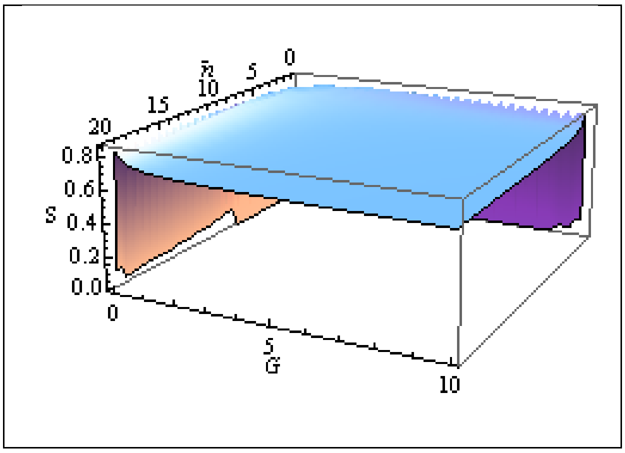


Fig. 8. shows the Von-Neumann entropy of the system with the saturation parameter and the mean photons number, which described an "unstable spiral" state, according to the eigenvalues

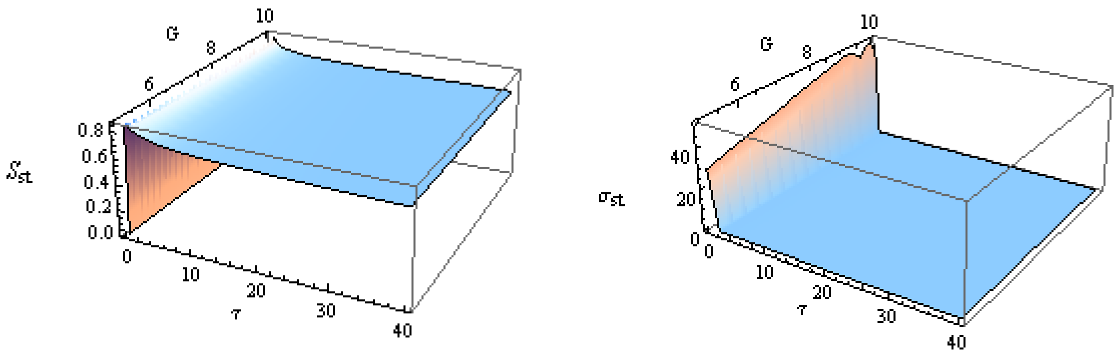


Fig. 9.a. in the case of strong field, shows the Von-Neumann entropy dependent on time and entropy production v.s. the saturation parameter and the time.

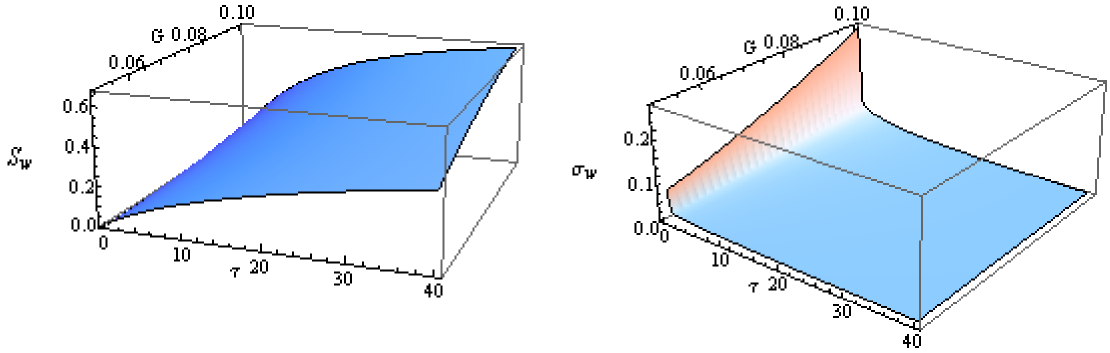


Fig. 9.b. shows the Von-Neumann entropy of the system and entropy production with the saturation parameter and the time in the case of the weak field

A standing wave or stationary wave consists of nodes and antinodes. The peak of the wave oscillation at any point in space is constant with time. This work discusses the radiation pressure on atoms, the velocity and the temperature. The radiation pressure on atoms is represented by the mean acting force with respect to the coherent state. This work presents two cases of detuning the first is larger than zero the other is smaller than zero. The change in the detuning value does not have any effect on the motion, but in case of the time dependence of the mean photon numbers the situation will change substantially. In this work a standing wave completes its period at a time of 1.53 duration for a distance of 0.5 cm. Now two cases about a standing wave will be introduced; the first 2-nodes and 3-antinodes, the second 3-nodes and 2-antinodes. The coherence angle plays an important role to determine the standing wave. The force makes 3-antinodes at the coherence angles $\phi = \{\frac{\pi}{4}, \frac{3\pi}{4}\}$, and it makes 2-antinodes at $\phi = \{\frac{\pi}{2}, \pi\}$, as shown in Fig. (10).

The velocity represented at the same angles also is forming a standing wave, it perverse the same behavior as in the case of the force. It forms 3-antinodes at the- coherence angles $\phi = \{\frac{\pi}{2}, \pi\}$, forms 2-antinodes at $\phi = \{\frac{\pi}{4}, \frac{3\pi}{4}\}$ as shown in Fig. (11).

In the course of time the pattern of “Egg Crate” appears, see figure Fig. (12.a) for the pressure force, and Fig. (12.b) represents the velocity of the atoms.

9.3 Two Cases Arise Concerning the Spontaneous Emission

9.3.1 Slow spontaneous emission

In the case of the strong field, and the relation between the time and the mean number of photons is $\bar{n} = 0.5\tau$, in which the saturation parameter is larger than 1. The force is measured by the unit of Newton. Considering a distance 1 cm and time 1.53, the rate of change in their force change from [-2 * 10^-14 N] to [2* 10^-14 N]. The waveform shows periodical structure forming repeated nonlinear layers.

The resulting figures show a standing wave with nodes and antinodes with the change of time. This behavior is similar to the potential function, see Fig. 13.

If the time of the experiment is increased to 3.5, as in Fig. 14, so more changes with time will occur, showing an increase in amplitude as time increases. The figures are also consistent with the potential form, “Egg Crate” pattern is introduced.

The same effect appears in the nonlinear amplitude as that it is increases with increasing time to 6, and still controlled by the periodicity, see Fig. 15.

Here, the effect of the change in distance to 2 cm appears most clearly in the form of a standing wave in repeated parallel layers. The shown velocities represent the difference between the vapor velocity before interaction and its velocity after interaction.

The change in speed by the change in the time is shown as a standing wave with an amplitude modulator wave, see Fig. 16.

The left hand side figures from Fig. 13 to Figs. 16 represent nonlinear standing waves with nodes and anti-nodes, and their right side figures show the solutions like an "Egg Crate" pattern.

As the time changes to 6 and the distance to 2 cm, Fig. 17.a, the speed appears as a periodic function with peaks and bottoms, which looks like the patters in Fig. 17.b, this agrees with the study in [23].

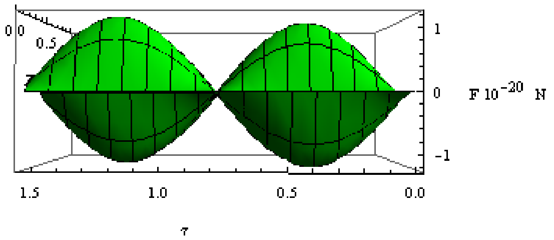


Fig. 10 a. Standing wave with 2-antinode ,and 3-nodes

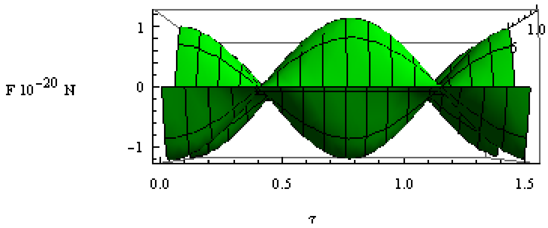
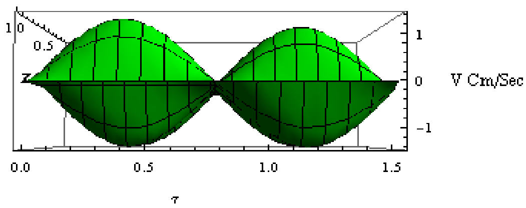


Fig. 10 b. Standing wave with 3-antinode,and 2-nodes



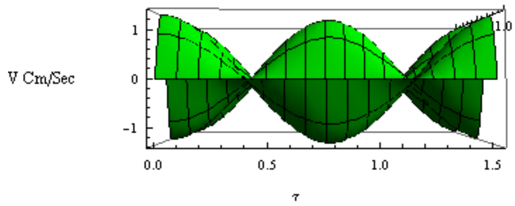


Fig. 11. Standing wave with 3 antinodes

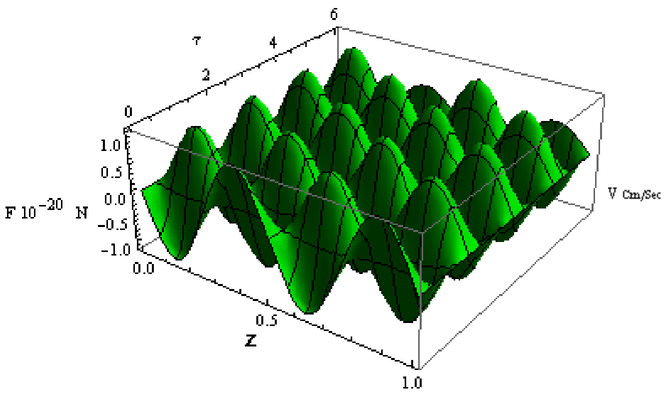


Fig. (12.a)

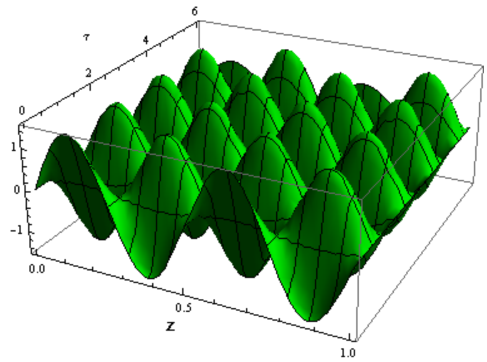


Fig. (12.b)

Fig. 12. An Egg Crate structure

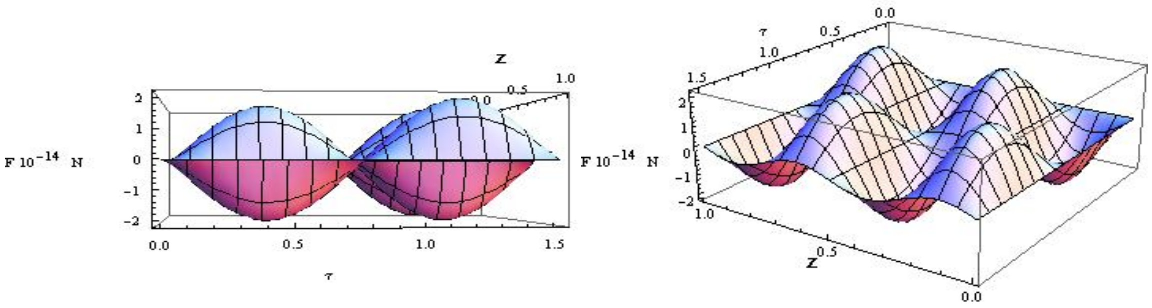


Fig. 13. Potential function

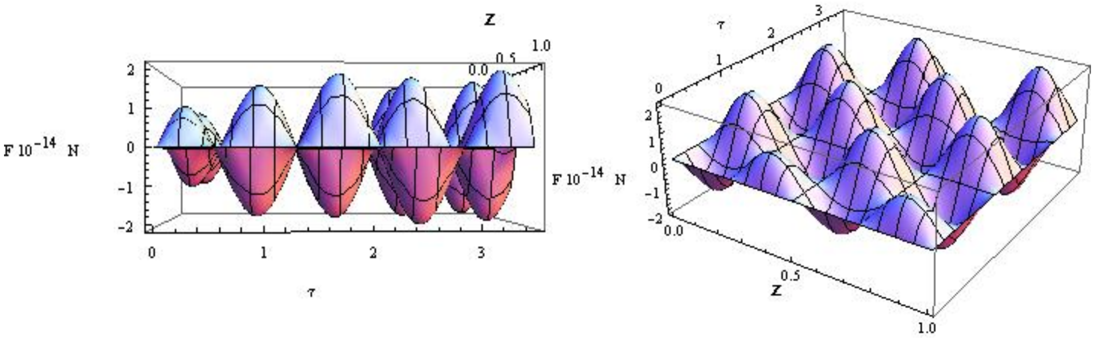


Fig. 14. Potential function increased to 3.5 seconds

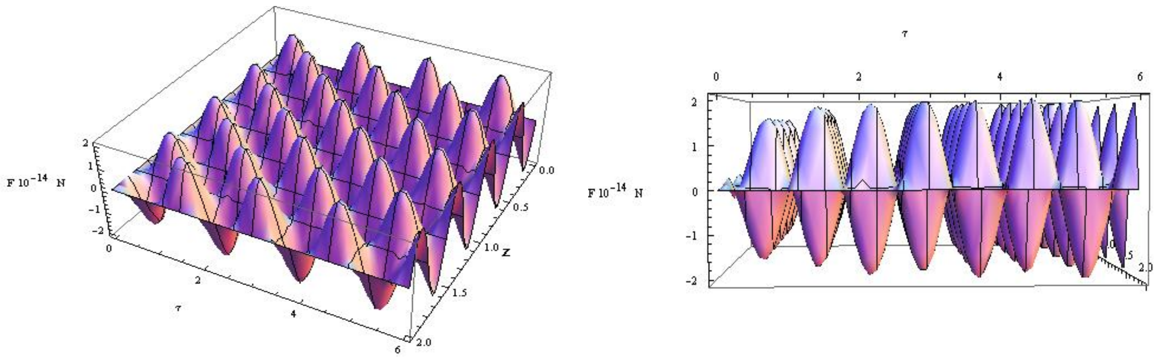


Fig. 15. periodic function

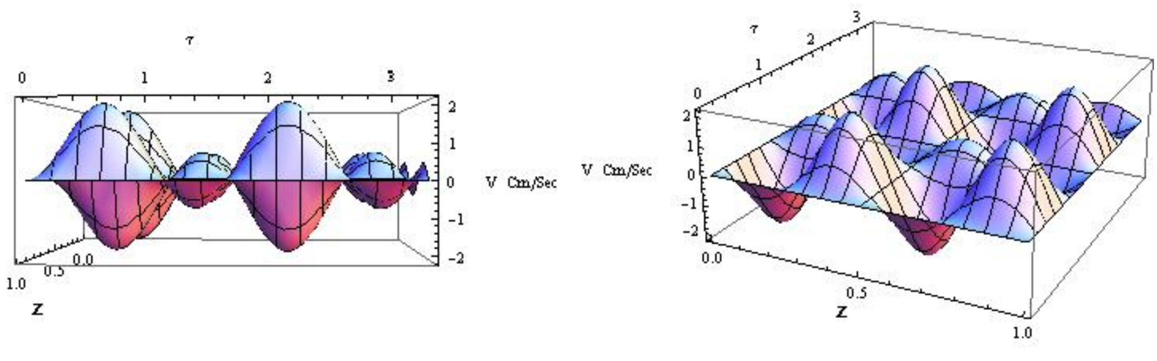
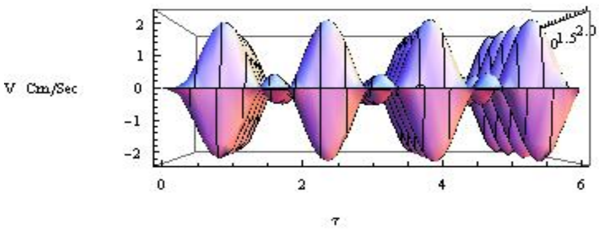


Fig. 16. Amplitude of modulator wave



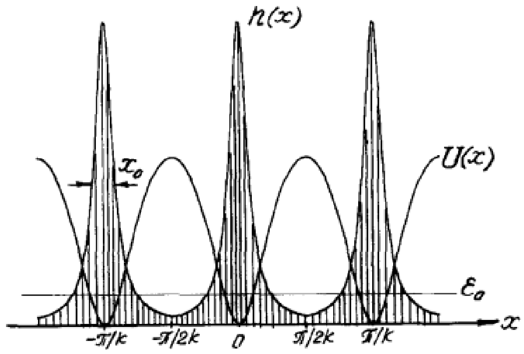
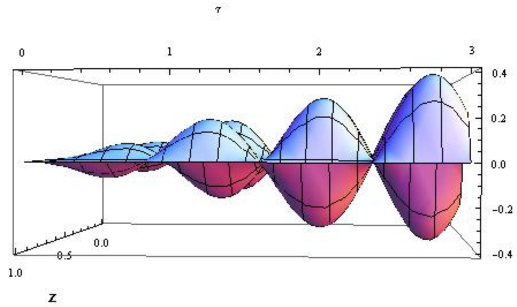
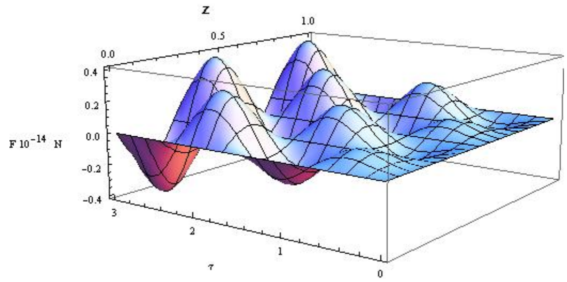


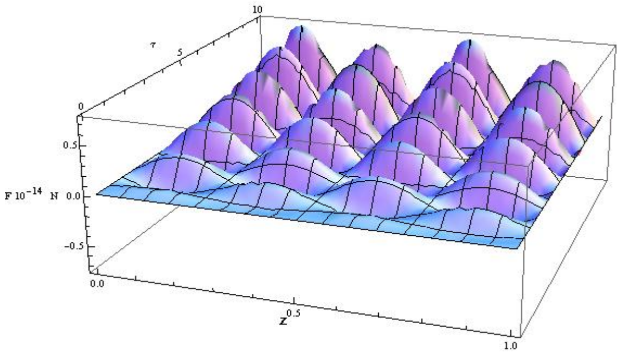
Fig. (17.a) **Fig. (17.b)**

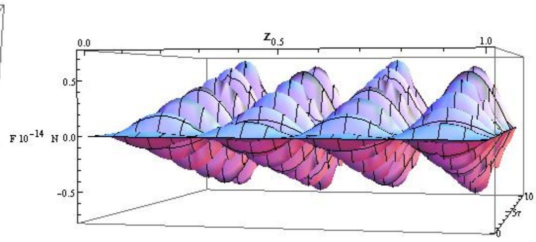
Fig. 17. periodic function with peaks and bottoms



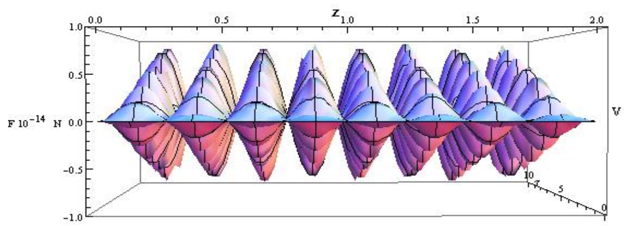


Figs. 18. Amplitude variation





Figs. 19. Standing wave due to nodes



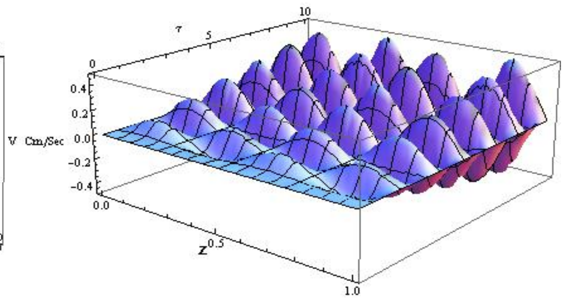


Fig. (20.a) : standing wave due to nodes **Fig. (20.b) standing wave due to antinodes**

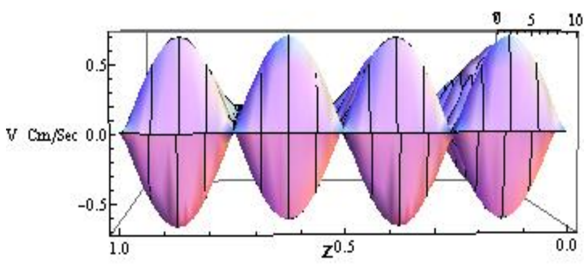


Fig. (21,a)

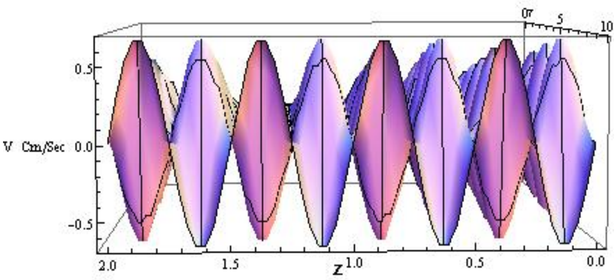


Fig. (21,b)

Fig. 21. Distance increase wave

9.3 Rapid Spontaneous Emission (Weak Field)

For the weak field i.e. $\bar{n} = 2G\tau$, as the time changes and for a fixed and low coefficient of saturation and different coherent angles of the laser, the force behavior is show in Fig. 18. The force reflects the pressure radiation on the atoms in unit area in the form of periodic interference waves taking the form of nonlinear standing waves with nodes and antinodes, their amplitudes vary in increasing tendency with distance and fluctuating in the interval (-0.4*10^-14 N, 0.4 *10^-14 N). It also appears as "Egg Crate" pattern.

In Figs. 19, the view angles focus on space travelled by the waves due to interference, which appear as standing nonlinear waveforms in parallel layers with increasing amplitudes. When the time changes in the period up to 10 in dimensionless scale, the number of nodes and antinodes increases with time with increasing amplitudes. Also "Egg Crate" form is revealed.

When the distance changes to 2 cm, the number of nodes and antinodes increases along the time axis in increasing amplitude for the pressure force, as shown in Fig. 20.a. The velocity of atoms has the same behavior as in Fig. (20.b). Both looks like an "Egg- Crate" from different view angles.

Along the increasing distance intervals for the atoms velocity, the standing wave behavior appears in harmonic growing with time wave packets, as in Fig. 21a,b.

10. CONCLUSION

The study is conducted using sodium atoms vapor [24] in the framework of the quantum Dirac picture, focusing on the role played by the coherence angles of the laser mode. This study is a reviewing and verification of the previous work in [4]. Taking into consideration that the collision between atoms is neglected. The steady state of the optical Bloch equations for the density matrix elements are evaluated, then the mean force with respect to the coherent state causes the deceleration and acceleration and the change in detuning, taking in details the first two out of four cases:

1) $\vec{R} = -ke_z, \Delta > 0$.
2) $\vec{R} = ke_z, \Delta > 0$.
3) $\vec{R} = -ke_z, \Delta < 0$.
4) $\vec{R} = ke_z, \Delta < 0$.

Throughout the investigation of the problem, in terms of mean photon numbers it has been found that: in the classical approach the mean number of photons is dealt with as a continuous quantity according to [25, 26], which changes within the interval [0,20]. The interaction angles are in terms of time and distance. The evolution in the behavior of atoms is investigated according to changing the angles of coherence through by applying optical filters, playing a principal role in inducing the deceleration or the acceleration of the atoms. It is kept in mind that, the atomic vapor is immersed in an energy bath presented by the laser field, such that the state of the atomic vapor is unstable inside the system due to the loss or gain of energy caused by the deceleration or acceleration of the atoms influenced by the change of the coherence angles. The investigation of the problem, according to the weak and strong spontaneous emission in terms of time dependent mean photon numbers, introduced by Kazantsev and Pusep, allows to determine the entropy production of the unstable system, and then focusing on the spatio-temporal behavior of atomic beam under the influence of the radiation pressure to follow the deceleration and the acceleration, showing the formation of nonlinear standing waveforms oscillating in Egg Crate patterns found in previous studies. Furthermore, it was concluded that when the force is collected in two opposite directions, there is no guided wave of laser waves, which is contrary to the Stenholm imposition.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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