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Mathematical Modelling for Semiconductor and Piezoelectric Media

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Author's contribution

The sole author designed, analyzed, interpreted and prepared the manuscript.

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ABSTRACT

In this analysis the importance of mathematical modelling of the physical systems has been outlined. The constitutive relations and basic governing equations of motion for homogeneous isotropic elastic semiconductor (n-type) and homogeneous transversely isotropic ($6mm$ class) piezoelectric elastic media, in the absence of body forces and electric sources are made non-dimensional in order to reduce the mathematical complexity. All the obtained equations are rewritten in matrix form. Then considering the harmonic wave solution the eigen values and eigen vectors are calculated to obtained the formal solution of the problem.

Keywords: Piezoelectric; semiconductor; transversely isotropic; cardano method.

1. INTRODUCTION

Many authors [1-8] have studied the surface waves in piezoelectric, semiconductor as well as in layered structures. Sharma et al. [5-7] modelled the surface waves in piezoelectric and semiconductor layered structures well. It is obvious that no physical system can be represented in its full physical details and therefore idealizing assumptions are made for the purpose of analysis and synthesis of the system. Once a physical system is obtained, the next step is to obtain a mathematical model which is the mathematical representation of

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physical model through use of appropriate physical laws. Therefore, this paper has arisen from the efforts to formulate a simple unified approach to system modelling. In particular, the aim was to develop modelling in such a way that it would complement dynamical systems analysis and control system studies. The modern heavily theoretical approach to control systems analysis and design has caused a decline in the intuitive understanding of engineering systems and the way in which they behave. So, the system modelling methods have been developed to establish this understanding and hence to find the fundamental properties of the physical system.

Mathematical models are extremely powerful because they usually enable predictions to be made about a system. These predictions provide a road map for further experimentations. Here an attempt is made to model a physical system mathematically to control and explain it. The physical system is interpreted as mathematical abstraction in terms of a set of differential equations. These equations are made non-dimensional then solved by employing functional iteration method along with irreducible Cardano method. This feature makes the problem interesting also from mathematical point of view. These equations are then modelled to specify the solution for the problems to be undertaken for investigation. The formal solution of the resulting model has also been developed here which can be conveniently used in solving the various problem related to the materials.

2. BASIC EQUATIONS AND CONSTITUTIVE RELATIONS

In this section we will outline the constitutive relations and basic governing equations of motion for homogeneous isotropic elastic semiconductor (n-type) and homogeneous transversely isotropic ($6mm$ class) piezoelectric elastic media, in the absence of body forces and electric sources.

2.1 Semiconductor Materials

The geometric and constitutive relations, equations of motion and electron diffusion in an elastic semiconductor (n-type) material in the absence of thermal field are reported below [4-6]:
Strain-displacement relations

$$S_{ij}^s = \left(u_{i,j}^s + u_{j,i}^s\right)/2\,,\ i,\ j = 1,\ 2,\ 3 \qquad (1)$$

Sress-strain-electron concentration relation

$$\tau_{ij}^s = 2\mu S_{ij}^s + \left(\lambda S_{kk}^s + \lambda^n N\right)\delta_{ij}\ i,\ j = 1,\ 2,\ 3 \qquad (2)$$

Current density

$$J_z^s = -eD^n N_{,z} \qquad (3)$$

Equations of motion

$$\tau_{ij,j} = \rho^s \ddot{u}_i^s\,, i,\ j = 1,\ 2,\ 3 \qquad (4)$$

Equation of electron diffusion

$$\rho^s D^n N_{,ii} - \rho^s\left(\dot{N} + t^n \ddot{N}\right) - a_2^n T_0 \lambda^T \dot{u}_{i,i}^s = \frac{\rho^s}{t_n^+}\left(N + t^n \dot{N}\right)$$
$$, i = 1,\ 2,\ 3 \qquad (5)$$

Where

$$N = n - n_0$$

$$a_2^n = \frac{a^{Qn}}{a^n}$$

$$\lambda^T = \left(3\lambda + 2\mu\right)\alpha_T$$

and $u_i^s\ \left(i = 1,\ 2,\ 3\right)$ are the components of the displacement vector $\mathbf{u^s} = \left(x_1,\ x_2\ x_3\ t\right)$ in semiconductor material. Here the quantities are the stresses $\lambda,\ \mu$ are Lamè's parameters; ρ^s is the density; λ^n is the elastodiffusive constant of electrons; D^n is the diffusion coefficient of electron; t_n^+ and t^n are, respectively the life time and relaxation time of the carriers fields; n_0 and n are the equilibrium and non-equilibrium values of electrons concentration; α_T is the coefficient of linear thermal expansion of the semiconductor material. The quantities $a^{Qn},\ a^n$ are flux-like constants, T_0 is the uniform temperature of the semiconductor. The quantities $\tau_{ij}^s\ \left(i,\ j = 1,\ 2,\ 3\right)$, J_z^s and e represent the stresses, current density and electronic charge for the semiconductor material. The superposed dots on various quantities denote time differentiation and comma notation is used for spatial derivatives. The sum is taken over the repeated indices unless stated otherwise.

2.2 Piezoelectric Materials

The geometric and constitutive relations, equation of motion and Gauss equation for piezoelectric material are outlined here in the absence of thermal field as below:

Strain-displacement relations [7-9]

$$S_{ij}^{p} = \left(u_{i,j}^{p} + u_{j,i}^{p}\right)/2 \; i,\ j = 1,\ 2,\ 3 \tag{6}$$

Constitutive Relations

(i) Stress-strain-electric field relations

$$\tau_{ij}^{p} = c_{ijkl} S_{kl}^{p} - e_{kij} E_{k}\ , i,\ j = 1,\ 2,\ 3 \tag{7}$$

(ii) Electric displacement-strain-electric field relations:

$$D_{i}^{p} = e_{ijk} S_{jk}^{p} + \varepsilon_{ij} E_{j}\ , i,\ j = 1,\ 2,\ 3 \tag{8}$$

Equations of motion

$$\tau_{ij,j}^{p} = \rho^{p} \ddot{u}_{i}^{p}\ \ , i,\ j = 1,\ 2,\ 3 \tag{9}$$

Gauss equation for piezoelectrics is the electric field, ϕ^{p} denotes the electric potential and $u_{i}^{p}\ (1,\ 2,\ 3)$ are the components of displacement vector $\mathbf{u^{p}} = (x_1,\ x_2\ x_3\ t)$ for piezoelectric material. The quantities ρ^{p}, c_{ij} and e_{ij} are the density, elastic parameters and piezoelectric constants; ε_{11} and ε_{33} are the electric permittivity perpendicular and along the axis of symmetry of piezoelectric material, respectively. The quantity $\tau_{ij}^{p}\ (i,\ j = 1,\ 2,\ 3)$ represents the stresses for piezoelectric material. Throughout the thesis the superscripts p and s on the field quantities as well as material parameters refer to piezoelectric and semiconductor materials respectively. The summation over the repeated indices prevails unless stated otherwise.

$$D_{i,i}^{p} = 0 \tag{10}$$

where

$$E_{i} = -\phi_{,i}^{p} \tag{11}$$

3. DEVELOPMENT OF MATHEMATICAL MODEL

We take the rectangular cartesian coordinate system $O(x_1,\ x_2,\ x_3)$ as $O(x,\ y,\ z)$ and restrict our investigations to two-dimensional structures, in the $x - z$ plane. Thus, the basic governing equations of motion, electron diffusion and Gauss equation for semiconductor and piezoelectric media are given as under:

Semiconductor

The equations (4) and (5) satisfied by the displacement vector $\mathbf{u^{s}}(x,\ z,\ t) = (u^{s},\ 0,\ w^{s})$ and electron concentration $N(x,\ z,\ t)$ for two-dimensional semiconductor medium in vector form become:

$$\mu \nabla^2 \mathbf{u^{s}} + (\lambda + \mu)\nabla\nabla \cdot \mathbf{u^{s}} - \lambda^{n} \nabla N = \rho^{s} \ddot{\mathbf{u}}^{\mathbf{s}} \tag{12}$$

$$\rho^{s} D^{n} \nabla^{2} N - \rho^{s}\left(1 + t^{n}\frac{\partial}{\partial t}\right)\dot{N} - a_2^{n} T_0 \lambda^{T} \nabla \cdot \dot{\mathbf{u}}^{\mathbf{s}} = -\left(1 + t^{n}\frac{\partial}{\partial t}\right)\left(\frac{\rho^{s}}{t_n^{+}}\right)N \tag{13}$$

Piezoelectric

The equations (9) and (10) satisfied by the displacement vector $\mathbf{u^{p}}(x,\ z,\ t) = (u^{p},\ 0,\ w^{p})$ and electric potential $\phi^{p}(x,\ z,\ t)$ for two-dimensional piezoelectric elastic medium become:

$$c_{11} u_{,xx}^{p} + c_{44} u_{,zz}^{p} + (c_{13} + c_{44}) w_{,xz}^{p} + (e_{15} + e_{31}) \phi_{,xz}^{p} = \rho^{p} \ddot{u}^{p} \tag{14}$$

$$(c_{13} + c_{44}) u_{,xz}^{p} + c_{44} w_{,xx}^{p} + c_{33} w_{,zz}^{p} + e_{15} \phi_{,xx}^{p} + e_{33} \phi_{,zz}^{p} = \rho^{p} \ddot{w}^{p} \tag{15}$$

$$(e_{15}+e_{31})u^p_{,xz}+e_{15}w^p_{,xx}+e_{33}w^p_{,zz}-\varepsilon_{11}\phi^p_{,xx}-\varepsilon_{33}\phi^p_{,zz}=0 \tag{16}$$

In order to simplify the system of equations (12)-(16) and facilitate their solution, we introduce the following non-dimensional quantities:

$$x'=\frac{\omega^* x}{v_l} \quad z'=\frac{\omega^* z}{v_l} \quad t'=\omega^* t$$

$$u^{s'}=\frac{\rho^s\omega^* v_l}{\lambda^n n_0}u^s \quad w^{s'}=\frac{\rho^s\omega^* v_l}{\lambda^n n_0}w^s \quad N'=\frac{N}{n_0}$$

$$u^{p'}=\frac{\rho^s\omega^* v_l}{\lambda^n n_0}u^p \quad w^{p'}=\frac{\rho^s\omega^* v_l}{\lambda^n n_0}w^p \quad \phi^{p'}=\varepsilon_p\phi^p$$

$$\tau^{s'}_{ij}=\frac{\tau^s_{ij}}{\lambda^n n_0} \quad J^{s'}_z=\frac{J^s_z}{e n_0 v_l} \quad \varepsilon_n=\frac{a_2^n\lambda^{T^2}T_0^2\overline{\lambda}_n}{\rho^s(\lambda+2\mu)n_0}$$

$$\tau^{p'}_{ij}=\frac{\tau^p_{ij}}{\lambda^n n_0} \quad D^{p'}_z=\frac{\rho^s v_l^2}{\lambda^n n_0 e_{33}}D^p_z \quad \varepsilon_p=\frac{e_{33}\omega^*\rho^s v_l}{c_{11}\lambda^n n_0}$$

$$\overline{\lambda}_n=\frac{\lambda^n n_0}{\lambda^T T_0} \qquad \delta_1^2=\frac{v_l^2}{v_p^2} \qquad k'=\frac{k v_l}{\omega^*}$$

$$\overline{\rho}=\frac{\rho^p}{\rho^s} \quad \eta_3=\frac{\varepsilon_{33}c_{11}}{e_{33}^2} \quad \omega'=\frac{\omega}{\omega^*} \quad h'=\frac{\omega^* h}{v_l}$$

$$c_1=\frac{c_{33}}{c_{11}} \qquad c_2=\frac{c_{44}}{c_{11}} \qquad c_3=\frac{c_{13}+c_{44}}{c_{11}}$$

$$e_1=\frac{e_{15}+e_{31}}{e_{33}} \qquad e_2=\frac{e_{15}}{e_{33}} \qquad \overline{\varepsilon}=\frac{\varepsilon_{11}}{\varepsilon_{33}}$$

$$t^{n'}=\omega^* t^n \quad t_n^{+'}=\omega^* t_n^+ \tag{17}$$

Where we have defined the quantities

$$v_l^2=\frac{\lambda+2\mu}{\rho^s} \quad v_t^2=\frac{\mu}{\rho^s}$$

$$v_p^2=\frac{c_{11}}{\rho^p}$$

$$\omega^*=\frac{v_l^2}{D^n} \tag{18}$$

Here ω^*, v_l and v_t are respectively the characteristic frequency, longitudinal and shear wave velocities in semiconductor and v_p denotes the longitudinal wave velocity in piezoelectric material.

The non-dimensional equation for semiconductor and piezoelectric continuum are obtained as below:

Semiconductor

Upon using the quantities (17) and (18) in equations and (12) - (13) we obtain

$$\delta^2\nabla^2\mathbf{u}^{\mathbf{s}} + \left(1-\delta^2\right)\nabla\nabla\cdot\mathbf{u}^{\mathbf{s}} - \nabla N = \ddot{\mathbf{u}}^{\mathbf{s}} \quad (19)$$

$$\nabla^2 N - \left[-\frac{1}{t_n^+} + \left(1-\frac{t^n}{t_n^+}\right)\frac{\partial}{\partial t} + t^n\frac{\partial^2}{\partial t^2}\right]N - \varepsilon_n\nabla\cdot\dot{\mathbf{u}}^{\mathbf{s}} = 0 \quad (20)$$

The constitutive relations (2.2) and (2.3) in non-dimensional form become:

$$\tau_{zz}^s = \left(1-2\delta^2\right)\frac{\partial u^s}{\partial x} + \frac{\partial w^s}{\partial z} - N \quad (21)$$

$$\tau_{xz}^s = \delta^2\left(\frac{\partial u^s}{\partial z} + \frac{\partial w^s}{\partial x}\right) \quad (22)$$

$$J_z^s = -\frac{\partial N}{\partial z} \quad (23)$$

We introduce the scalar and vector point potential functions ϕ^s and ψ^s through the relations

$$u^s = \frac{\partial\phi^s}{\partial x} + \frac{\partial\psi^s}{\partial z}$$

$$w^s = \frac{\partial\phi^s}{\partial z} - \frac{\partial\psi^s}{\partial x} \quad (24)$$

In order to facilitate the solutions in semiconductor continuum.

Using relations (24) in system of equations (19) and (20), we obtain

$$\nabla^2\phi^s - N - \ddot{\phi}^s = 0 \quad (25)$$

$$\nabla^2\psi^s = \frac{\ddot{\psi}^s}{\delta^2} \quad (26)$$

$$\nabla^2 N - \left[-\frac{1}{t_n^+} + \left(1+\frac{t^n}{t_n^+}\right)\frac{\partial}{\partial t} + t^n\frac{\partial^2}{\partial t^2}\right]N - \varepsilon_n\nabla^2\dot{\phi}^s = 0 \quad (27)$$

Substituting expressions (24) in equations (21)-(22) and using equations (25)-(26) we get

$$\tau_{zz}^s = \ddot{\phi}^s - 2\delta^2\left(\frac{\partial^2\phi^s}{\partial x^2} + \frac{\partial^2\psi^s}{\partial x\partial z}\right) \quad (28)$$

$$\tau_{xz}^s = \ddot{\psi}^s + 2\delta^2\left(\frac{\partial^2\phi^s}{\partial x\partial z} - \frac{\partial^2\psi^s}{\partial x^2}\right) \quad (29)$$

The equation (26) corresponds to purely transverse waves in the semiconductor. This gets decoupled from rest of the motion and is not affected by the charge carrier fields.

Piezoelectric

Upon using the quantities (17) and (18) in equations and (14) - (16) we obtain

$$u_{,xx}^p + c_2u_{,zz}^p + c_3w_{,xz}^p + e_1\phi_{,xz}^p = \delta_1^2\ddot{u}^p \quad (30)$$

$$c_3u_{,xz}^p + c_2w_{,xx}^p + c_1w_{,zz}^p + e_2\phi_{,xx}^p + \phi_{,zz}^p = \delta_1^2\ddot{w}^p \quad (31)$$

$$e_1u_{,xz}^p + e_2w_{,xx}^p + w_{,zz}^p - \eta_3\bar{\varepsilon}\phi_{,xx}^p - \eta_3\phi_{,zz}^p = 0 \quad (32)$$

The constitutive relations (7) and (8) in non-dimensional form become:

$$\tau_{zz}^p = \frac{\bar{\rho}}{\delta_1^2}\left\{\left(c_3-c_2\right)\frac{\partial u^p}{\partial x} + c_1\frac{\partial w^p}{\partial z} + \phi_{,z}^p\right\} \quad (33)$$

$$\tau_{xz}^p = \frac{\bar{\rho}}{\delta_1^2}\left\{\frac{c_2}{2}\left(\frac{\partial u^p}{\partial z} + \frac{\partial w^p}{\partial x}\right) + e_2\phi_{,x}^p\right\} \quad (34)$$

$$D_z^p = \left(e_1-e_2\right)\frac{\partial u^p}{\partial x} + \frac{\partial w^p}{\partial z} - \eta_3\phi_{,z}^p \quad (35)$$

The equations (25)-(27), (28)-(29) and (23) for semiconductor medium and equations (30)-(35) for piezoelectric continuum can be rewritten in the matrix form as given below:

Semiconductor

$$\mathbf{H}^*\mathbf{Z} = 0 \quad (36)$$

$$\boldsymbol{\tau}^{\mathbf{s}} = \hat{\mathbf{H}}^*\mathbf{Z} \quad (37)$$

Piezoelectric

$$\mathbf{G}^*\mathbf{Y} = 0 \quad (38)$$

$$\boldsymbol{\tau}^{\mathbf{p}} = \hat{\mathbf{G}}^{*}\mathbf{Y} \tag{39}$$

Where

$$\mathbf{Z} = \begin{bmatrix} \phi^{s} & N & \psi^{s} \end{bmatrix}^{T} \tag{40}$$

$$\mathbf{Y} = \begin{bmatrix} u^{p} & w^{p} & \phi^{p} \end{bmatrix}^{T} \tag{41}$$

$$\boldsymbol{\tau}^{\mathbf{s}} = \left[\tau_{zz}^{s},\, \tau_{xz}^{s},\, J_{z}^{s}\right]^{T} \tag{42}$$

$$\boldsymbol{\tau}^{\mathbf{p}} = \left[\tau_{zz}^{p},\, \tau_{xz}^{p},\, D_{z}^{p}\right]^{T} \tag{43}$$

$$\hat{\mathbf{H}}^{*} = \begin{bmatrix} \frac{\partial^2}{\partial t^2} - 2\delta^2 \frac{\partial^2}{\partial x^2} & -2\delta^2 \frac{\partial^2}{\partial x \partial z} & 0 \\ 2\delta^2 \frac{\partial^2}{\partial x \partial z} & \frac{\partial^2}{\partial t^2} - 2\delta^2 \frac{\partial^2}{\partial x^2} & 0 \\ 0 & 0 & -\frac{\partial}{\partial z} \end{bmatrix} \tag{44}$$

$$\mathbf{G}^{*} = \begin{bmatrix} \frac{\partial^2}{\partial x^2} + c_2 \frac{\partial^2}{\partial z^2} - \delta_1^2 \frac{\partial^2}{\partial t^2} & c_3 \frac{\partial^2}{\partial x \partial z} & e_1 \frac{\partial}{\partial x \partial z} \\ c_3 \frac{\partial^2}{\partial x \partial z} & c_2 \frac{\partial^2}{\partial x^2} + c_1 \frac{\partial^2}{\partial z^2} - \delta_1^2 \frac{\partial^2}{\partial t^2} & e_2 \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \\ e_1 \frac{\partial^2}{\partial x \partial z} & e_2 \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} & -\eta_3 \left(\bar{\varepsilon} \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) \end{bmatrix}$$

$$\hat{\mathbf{G}}^{*} = \frac{\bar{\rho}}{\delta_1^2} \begin{bmatrix} (c_3 - c_2)\frac{\partial}{\partial x} & c_1 \frac{\partial}{\partial z} & \frac{\partial}{\partial z} \\ \frac{c_2}{2}\frac{\partial}{\partial z} & \frac{c_2}{2}\frac{\partial}{\partial x} & e_2 \frac{\partial}{\partial x} \\ (e_1 - e_2)\frac{\delta_1^2}{\bar{\rho}}\frac{\partial}{\partial x} & \frac{\delta_1^2}{\bar{\rho}}\frac{\partial}{\partial z} & -\eta_3 \frac{\delta_1^2}{\bar{\rho}}\frac{\partial}{\partial z} \end{bmatrix} \tag{45}$$

$$\alpha_n' = -\frac{1}{t_n^{+}} + \left(1 + \frac{t^n}{t_n^{+}}\right)\frac{\partial}{\partial t} + t^n \frac{\partial^2}{\partial t^2} \tag{46}$$

The equations (36) and (38) are basic governing equations in matrix form whereas equations (37) and (39) are the constitutive relations for the semiconductor and piezoelectric media, respectively.

4. FORMAL SOLUTION OF THE MODEL

Suppose the time harmonic waves propagating along x-direction, so we consider solution of the form

$$\mathbf{Z}(x,\, z,\, t) = \overline{\mathbf{Z}}(z)\exp\{ik(x - ct)\} \tag{47}$$

$$\mathbf{Y}(x,\, z,\, t) = \overline{\mathbf{Y}}(z)\exp\{ik(x - ct)\} \tag{48}$$

Where the column vectors $\vec{\mathbf{Z}}$ and $\vec{\mathbf{Y}}$ are given by

$$\overline{\mathbf{Z}} = \left[\overline{\phi}^{\,s},\ \overline{N},\ \overline{\psi}^{\,s}\right]^T$$
$$\overline{\mathbf{Y}} = \left[\overline{u}^{\,p}, \overline{w}^{\,p}, \overline{\phi}^{\,p}\right]^T \tag{49}$$

Upon using solution (2.47) and (2.48) in equations (2.36) and (2.38) we have

Semiconductor

$$\left(\mathbf{H}_1 D^2 - \mathbf{H}_2\right)\overline{\mathbf{Z}} = 0 \tag{50}$$

Piezoelectric

$$\left(\mathbf{G}_1 D^2 + \mathbf{G}_2 D - \mathbf{G}_3\right)\overline{\mathbf{Y}} = 0 \tag{51}$$

Where

$$D^n \equiv \frac{d^n}{dz^n}, \qquad n = 1,\, 2$$

$$\mathbf{G_1} = \begin{bmatrix} c_2 & 0 & 0 \\ 0 & c_1 & 1 \\ 0 & 1 & -\eta_3 \end{bmatrix}$$

$$\mathbf{G_2} = ik\begin{bmatrix} 0 & c_3 & e_1 \\ c_3 & 0 & 0 \\ e_1 & 0 & 0 \end{bmatrix}$$

$$\mathbf{G_3} = k^2\begin{bmatrix} 1-\delta_1^2 c^2 & 0 & 0 \\ 0 & c_2 - \delta_1^2 c^2 & e_2 \\ 0 & e_2 & -\overline{\varepsilon}\eta_3 \end{bmatrix} \tag{52}$$

$$\mathbf{H_1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ ikc\varepsilon_n & 1 & 0 \end{bmatrix}$$

$$\mathbf{H_2} = \begin{bmatrix} 1-c^2 & -1 & 0 \\ 0 & 0 & 1-\dfrac{c^2}{\delta^2} \\ ikc\varepsilon_n & 1-\alpha_n^* c^2 & 0 \end{bmatrix} \tag{53}$$

$$\alpha_n^* = t^n + i\omega^{-1}\left(1 + \frac{t^n}{t_n^+}\right) + \frac{1}{t_n^+\omega^2}$$

Because the matrices $\mathbf{H_1}$ and $\mathbf{G_1}$ are non-singular, so the equations (50) and (51) can also be rewritten as

Semiconductor

$$\left(\mathbf{I}D^2 - \overline{\mathbf{H}}\right)\overline{\mathbf{Z}} = 0 \tag{54}$$

Piezoelectric

$$\left(\mathbf{I}D^2 + \overline{\mathbf{G}}_1 D - \overline{\mathbf{G}}_2\right)\overline{\mathbf{Y}} = 0 \tag{55}$$

Where the matrices $\overline{\mathbf{G}}_{\mathbf{i}}\ (1,\, 2)$ and $\overline{\mathbf{H}}$ are obtained as

$$\overline{\mathbf{G}}_1 = \mathbf{G_1^{-1}G_2} = ik\begin{bmatrix} 0 & \dfrac{c_3}{c_1} & \dfrac{e_1}{e_2} \\ \dfrac{c_3\eta_3 + e_1}{1 + c_1\eta_3} & 0 & 0 \\ \dfrac{c_3 - c_1 e_1}{1 + c_1\eta_3} & 0 & 0 \end{bmatrix}$$

$$\overline{\mathbf{G}}_2 = \mathbf{G_1^{-1}G_3} = k^2\begin{bmatrix} \dfrac{1-\delta^2 c^2}{c_2} & 0 & 0 \\ 0 & \dfrac{e_2 + \eta_3\left(c_2 - \delta_1^2 c^2\right)}{1 + c_1\eta_3} & \dfrac{\eta_3\left(e_2 - \overline{\varepsilon}\right)}{1 + c_1\eta_3} \\ 0 & \dfrac{c_2 - \delta_1^2 c^2 - c_1 e_2}{1 + c_1\eta_3} & \dfrac{e_2 + \overline{\varepsilon}\eta_3 c_1}{1 + c_1\eta_3} \end{bmatrix} \tag{56}$$

$$\overline{\mathbf{H}} = \mathbf{H_1^{-1}H_2} = k^2 \begin{bmatrix} 1-c^2 & 0 & 0 \\ 0 & 0 & 1-\dfrac{c^2}{\delta^2} \\ ikc\varepsilon_n c^2 & 1-\alpha_n^* c^2 + ikc\varepsilon_n & 0 \end{bmatrix} \tag{57}$$

And $\mathbf{I}$ is an identity matrix of order 3.

The equations (54) and (55) define eigenvalue problems and hence their solution will be of the form

$$\overline{\mathbf{Z}} = \overline{\mathbf{H}}^*(k, c)e^{nz} \tag{58}$$

$$\overline{\mathbf{Y}} = \overline{\mathbf{G}}^*(k, c)e^{mz} \tag{59}$$

Where $\overline{\mathbf{G}}^*(k, c)$ and $\overline{\mathbf{H}}^*(k, c)$ are the eigenvectors corresponding to the eigenvalues m and n pairs in the respective medium.

Now we shall discuss the equations (54) and (58) as well as (55) and (59) one by one for different constituents as follows:

4.1 Piezoelectric Continuum

Upon using solution (59) in equation (55), we get

$$\left(m^2\mathbf{I} + m\overline{\mathbf{G}}_1 - \overline{\mathbf{G}}_2\right)\overline{\mathbf{G}}^*(k, c) = 0 \tag{60}$$

The equation (60) will have a non-trivial solution iff

$$\det\left(m^2\mathbf{I} + m\overline{\mathbf{G}}_1 - \overline{\mathbf{G}}_2\right) = 0 \tag{61}$$

Upon expanding equation (61) and simplifying, we get

$$m^6 - \xi_1 m^4 + \xi_2 m^2 - \xi_3 = 0 \tag{62}$$

Where

$$\xi_1 = k^2 \frac{c_2\chi_1 + (1+\eta_3 c_1)\chi_2 - c_3(c_3\eta_3 + 2e_2) + c_1 e_1^2}{c_2(1+\eta_3 c_1)}$$

$$\xi_2 = k^4 \frac{\left\{c_2\chi_3 + \chi_1\chi_2 - c_3(c_2\eta_3\bar{\varepsilon} + 2e_1e_2) + e_1^2(c_2 - \delta_1^2 c^2)\right\}}{c_2(1+\eta_3 c_1)}$$

$$\xi_3 = k^6 \frac{\chi_2\,\chi_3}{c_2(1+\eta_3 c_1)} \tag{63}$$

The quantities $\chi_i\ (i = 1, 2, 3)$ are given by

$$\chi_1 = \left(c_1\bar{\varepsilon} + c_2 - \delta_1^2 c^2\right)\eta_3 + 2e_2 \qquad \chi_2 = \left(1 - \delta_1^2 c^2\right) \qquad \chi_3 = \left(c_2 - \delta_1^2 c^2\right)\eta_3\bar{\varepsilon} + e_2^2 \tag{64}$$

The equation (62) being a cubic polynomial equation can be directly solved with the help of Cardano's method as follows:

Diminishing the roots of the equation (62) by $\xi_1/3$, we get

$$m^{*6} + 3Qm^{*2} + \Pi = 0 \tag{65}$$

Where

$$Q = -\frac{2}{27}\xi_1^3 + \frac{1}{3}\xi_1\xi_2 - \xi_3$$

$$\Pi = -\frac{1}{3}\xi_1^2 + \xi_2 \tag{66}$$

Let the roots of eq. (65) be given by

$$m^{*2} = \vartheta + \varpi \tag{67}$$

Cubing both sides

$$m^{*6} = (\vartheta + \varpi)^3 = \vartheta^3 + \varpi^3 - 3\vartheta\varpi(\vartheta + \varpi)$$

$$m^{*6} + 3\vartheta\varpi m^{*2} - \left(\vartheta^3 + \varpi^3\right) = 0$$

Comparing above with (65) we have

$$Q^3 = \vartheta^3\varpi^3$$

$$\Pi = -\left(\vartheta^3 + \varpi^3\right)$$

Thus ϑ and ϖ are the roots of the quadratic equation

$$t^2 + \Pi t + Q^3 = 0$$

Thus we have

$$\vartheta^3 = \frac{-\Pi + \sqrt{\Pi^2 - 4Q^3}}{2}$$

$$\varpi^3 = \frac{-\Pi - \sqrt{\Pi^2 - 4Q^3}}{2}$$

Now taking cube roots of these quantities and then using in eq. (67), we have

$$m^{*2} = \vartheta + \varpi \quad \vartheta\ell + \varpi\ell^2 \quad \vartheta\ell^2 + \varpi\ell \tag{68}$$

Where

$$\vartheta = \left[\frac{-\Pi + \sqrt{\Pi^2 - 4Q^3}}{2}\right]^{1/3}$$

$$\varpi = \left[\frac{-\Pi - \sqrt{\Pi^2 - 4Q^3}}{2}\right]^{1/3}$$

$$\ell = \frac{-1 + \sqrt{3}i}{2}$$

$$\ell^2 = \frac{-1 - \sqrt{3}i}{2}$$

Here 1, ℓ and ℓ^2 are the cube roots of unity. Thus the roots of equation (62) are given by

$$m_i^2 = m_i^{*2} + \frac{\xi_1}{3}, \quad i = 1, 2, 3 \tag{69}$$

The pair of roots m_i satisfying the property

$$m_4 = -m_1 \quad m_5 = -m_2 \quad m_6 = -m_3 \tag{70}$$

The solution (59) takes the form

$$\overline{\mathbf{Y}}(z) = \sum_{i=1}^{3}\left[\overline{\mathbf{G}}_{\mathbf{i}}^{*}(k, c)A_i^p \exp(-m_i z) + \overline{\mathbf{G}}_{\mathbf{i+3}}^{*}(k, c)A_{i+3}^p \exp(m_i z)\right] \tag{71}$$

Where

$$\overline{\mathbf{G}}_{\mathbf{i}}^{*} = \begin{bmatrix} 1 \\ M_i \\ P_i \end{bmatrix}, \quad i = 1, 2, 3, \qquad m = -m_i$$

$$\overline{\mathbf{G}}_{\mathbf{i+3}}^{*} = \begin{bmatrix} 1 \\ -M_i \\ -P_i \end{bmatrix}, \quad i = 1, 2, 3, \qquad m = m_i \tag{72}$$

$$M_i = ikm_i \left\{ c_3\eta_3 \left(m_i^2 - \bar{\varepsilon}k^2 \right) + e_1 \left(m_i^2 - e_2 k^2 \right) \right\} / \Delta$$

$$P_i = \frac{-ikm_i}{\Delta\eta_3 \left(m_i^2 - k^2\bar{\varepsilon} \right)} \left[e_1\Delta - \left(m_i^2 - e_2k^2 \right) \left\{ c_3\eta_3 \left(m_i^2 - \bar{\varepsilon}k^2 \right) + e_1 \left(m_i^2 - e_2k^2 \right) \right\} \right]$$

$$\Delta = \eta_3 \left\{ c_1 m_i^2 - k^2 \left(c_2 - \delta_1^2 c^2 \right) \right\} \left(m_i^2 - \bar{\varepsilon}k^2 \right) + \left(m_i^2 - e_2 k^2 \right)^2 \tag{73}$$

Thus, the formal solution for piezoelectric medium with the help of equations (48) and (71) becomes:

$$\mathbf{Y}(x, z, t) = \sum_{i=1}^{3} \left[\overline{\mathbf{G}}_{\mathbf{i}}^{*}(k, c) A_i^p \exp(-m_i z) + \overline{\mathbf{G}}_{\mathbf{i+3}}^{*}(k, c) A_{i+3}^p \exp(m_i z) \right] \exp\{ik(x - ct)\} \tag{2.74}$$

where $A_i^p \, (i = 1, 2, \ldots 6)$ are the arbitrary constants to be determined with the help of appropriate boundary conditions.

Using solution (74) in equation (39), the expressions for non-vanishing stresses and electric displacement are obtained as:

$$\boldsymbol{\tau}^{\mathbf{p}} = \sum_{i=1}^{3} \left[\hat{\mathbf{G}}_{\mathbf{i}}(k, c) \overline{\mathbf{G}}_{\mathbf{i}}^{*}(k, c) A_i^p \exp(-m_i z) + \hat{\mathbf{G}}_{\mathbf{i+3}}(k, c) \overline{\mathbf{G}}_{\mathbf{i+3}}^{*}(k, c) A_{i+3}^p \exp(m_i z) \right] \exp\{ik(x - ct)\} \tag{2.75}$$

$$\hat{\mathbf{G}}_{\mathbf{i}}(k, c) = \begin{bmatrix} ik(c_3 - c_2) & -m_i c_1 & -m_i \\ -\dfrac{m_i c_2}{2} & \dfrac{ikc_2}{2} & ike_2 \\ \dfrac{\delta_1^2}{\bar{\rho}} ik(e_1 - e_2) & -\dfrac{\delta_1^2}{\bar{\rho}} m_i & \dfrac{\delta_1^2}{\bar{\rho}} \eta_3 m_i \end{bmatrix} \tag{76}$$

Upon using equation (83) in equation (47) the formal solution for the semiconductor continuum is given as under

$$\mathbf{Z}(x, z\ t) = \sum_{i=1}^{3} \left[\overline{\mathbf{H}}_{\mathbf{i}}^{*}(k, c) A_i^s \exp(-n_i z) + \overline{\mathbf{H}}_{\mathbf{i+3}}^{*}(k, c) A_{i+3}^s \exp(n_i z) \right] \exp\{ik(x - ct)\} \tag{86}$$

Where $A_i^s \left(i = 1,\ 2, \ldots 6\right)$ are the arbitrary constants to be determined from the appropriate boundary conditions.

Using the solution (86) in equation (37), the expressions for non-vanishing stresses and current density are obtained as:

$$\boldsymbol{\tau}^{\mathbf{s}} = \sum_{i=1}^{3} \left[\hat{\mathbf{H}}_{\mathbf{i}}(k,\ c)\overline{\mathbf{H}}_{\mathbf{i}}^{*}(k,\ c)A_i^s \exp(-n_i z) + \hat{\mathbf{H}}_{\mathbf{i+3}}(k,\ c)\overline{\mathbf{H}}_{\mathbf{i+3}}^{*}(k,\ c)A_{i+3}^s \exp(n_i z)\right]\exp\{ik(x-ct)\} \quad (87)$$

Where

$$\hat{H}_i(k,\ c) = \begin{bmatrix} k^2\left(2\delta^2 - c^2\right) & 2\delta^2 ikn_i & 0 \\ -2\delta^2 ikn_i & k^2\left(2\delta^2 - c^2\right) & 0 \\ 0 & 0 & n_i \end{bmatrix} \quad (88)$$

The matrix $\hat{\mathbf{H}}_{\mathbf{i+3}}(k,\ c)$ can be written from the matrix (88) by replacing n_i with $-n_i$ therein.

The formal solutions given by (74) and (86) for piezoelectric and semiconductor media, respectively, can be used to investigate different wave propagation problems in various types of piezoelectric-semiconductor composite structures under different conditions/situations prevailing at the boundaries/interfaces of these structures.

5. CONCLUDING REMARKS

1. The constitutive relations and basic governing equations for semiconductor and piezoelectric continua have been outlined.
2. The governing equations and constitutive relations are made non-dimensional for mathematical convenience and to remove the complexity of dimensional analysis.
3. Assuming the wave solution for time harmonic plane waves propagating along x-axis, the systems of partial differential equations have been expressed as matrix differential equations in order to form an eigenvalue problem.
4. The complex cubic characteristic equation for the piezoelectric material has been solved by using irreducible cardano method to obtain the eigenvalues and corresponding eigenvectors.
5. The complex quadratic equation pertaining to semiconductor material provided the eigenvalues and eigenvectors for this medium.
6. The corresponding eigenvalues and eigenvectors so determined have been used to write the general solution of the model in respect of field quantities for both the media.
7. The formal solution for non-vanishing stresses, current density and electric displacement has also been derived.
8. The developed formal solution can be used in various types of piezoelectric-semiconductor layered structures under different conditions/situations prevailing at the boundaries/interfaces.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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