Current Journal of Applied Science and Technology

Volume 41, Issue 48, Page 82-90, 2022; Article no.CJAST.95476
ISSN: 2457-1024
(Past name: British Journal of Applied Science & Technology, Past ISSN: 2231-0843, NLM ID: 101664541)

Bayesian Estimation of Exponentiated Rayleigh Distribution under Symmetric and Asymmetric Loss Functions

Pavitra Kumari [a*], Vinay Kumar [a], Rohit Kundu [a] and Pardeep Kumar [a]

[a] *Department of Mathematics and Statistics, CCS Haryana Agricultural University, Hisar, India.*

Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

Article Information

DOI: 10.9734/CJAST/2022/v41i484036

Open Peer Review History:
This journal follows the Advanced Open Peer Review policy. Identity of the Reviewers, Editor(s) and additional Reviewers, peer review comments, different versions of the manuscript, comments of the editors, etc are available here: https://www.sdiarticle5.com/review-history/95476

Original Research Article

Received: 20/10/2022
Accepted: 28/12/2022
Published: 29/12/2022

ABSTRACT

In this paper, we have considered the estimation problem of one-parameter exponentiated Rayleigh distribution. The parameters are estimated using likelihood based inferential procedure. We have computed MLEs and Bayes estimates under informative and non-informative priors along with six different loss functions, the Bayes estimation was obtained “Squared error, Linear exponential, Precautionary, Entropy, De Groot and non-Linear exponential loss functions”. Finding a good estimator of the unidentified shape parameter is the study's main goal. The Bayesian estimates of the parameter of exponentiated Rayleigh distribution are obtained using Markov chain Monte Carlo (MCMC) simulation method. All the computations are performed in OpenBUGS and R software.

Keywords: Bayesian estimation; MLE; Bayes estimate; exponentiated Rayleigh distribution; loss function; prior; posterior.

**Corresponding author: E-mail: Yadavbunty67@gmail.com;*

1. INTRODUCTION

For modelling data, Burr [1] introduced twelve different forms of cumulative distribution function. Among them Burr Type X and Burr Type XII received the maximum attention, where Surles and Padgett [2] observed that the Burr Type X distribution (Exponentiated Rayleigh distribution) can be used quite effectively in modeling strength data and also modeling general lifetime data. Several aspects of the ER distribution were studied in literature, see foe example Sartawi and Abu-Salih [3], Jaheen [4], Ahmed, Fakhry and Jaheen [5], Karam and Jbur [6], Feroze and Aslam [7] and Sindhu and Aslam [8]. The cumulative distribution function (CDF) and the probability density function (pdf) of the ER distribution with shape parameter (θ > 0) are respectively as follows [9]:

Numerous authors have worked on the generalization of Rayleigh distribution; among them are Voda [10], Kundu and Raqab (2005), Raqab and Madi [11], Merovci [12], Dey et al. [13], Merovci and Elbatal [12], Ahmad et al. [14], Saima et al. [15], Ateeq et al. [16], Sofi et al. (2019).

$$F(x,\theta) = \left(1 - e^{-x^{-2}}\right)^{\theta} \quad ; \; x > 0; \; \theta > 0 \qquad (1)$$

$$f(x;\theta) = 2\theta x e^{x^2}(1 - e^{-x^2})^{(\theta-1)} ; \quad x > 0; \; \theta > 0 \qquad (2)$$

The random number has been generated by inverse transformation method, which is for uniform random U:

$$X_i = \left(-ln\left(1 - {U_i}^{1/\theta}\right)\right)^{0.5} ; \; i = 1,2, \ldots., n \qquad (3)$$

Studying a particular phenomenon via the challenge of estimating the unknown parameters in statistical distributions is one of the significant problems that individuals who are interested in applied statistics face. This study examines the estimations of the ER distribution's unknown shape parameter. The ER distribution is a significant distribution in operations research and statistics, and it is used in a number of fields including biology, agriculture, and other sciences [17,18]. Considering the Bayesian analysis of the unknown parameters with various priors (informative and non-informative) and loss functions for complete samples is the main goal of this [19,20].

2. LIKLEHOOD FUNCTION

Let X_1, X_2,..,X_n be a random sample from ER distribution with shape parameter θ >0. Therefore, the likelihood function of θ, from (1), as follows as:

$$L(\theta|data) = \prod_{i=1}^{n} f(x_i, \theta) \qquad (4)$$

$$L(\theta|data) = \prod_{i=1}^{n}\left(2\theta x e^{-x_i^2+\ln(1-x_i^2)^{-1}} e^{-\theta ln\left(1-e^{-x_i^2}\right)^{-1}}\right) \qquad (5)$$

$$L(\theta|data) = q_1\theta^n e^{-\theta[\sum_{i=1}^{n}\ln(1-e^{-x_i^2})^{-1}]} \qquad (6)$$

Such that

$$q_1 = 2^n e^{\left(\sum_{i=1}^{n} lnx_i - \sum_{i=1}^{n} x_{i=1}^2 + \sum_{i=1}^{n}\ln(1-e^{-x_i^2})^{-1}\right)}$$

2.1 Bayesian Estimators using Different Prior and Loss Functions

In this section Bayesian Estimators of the shape parameter for four different prior functions and under six different loss functions has been determined.

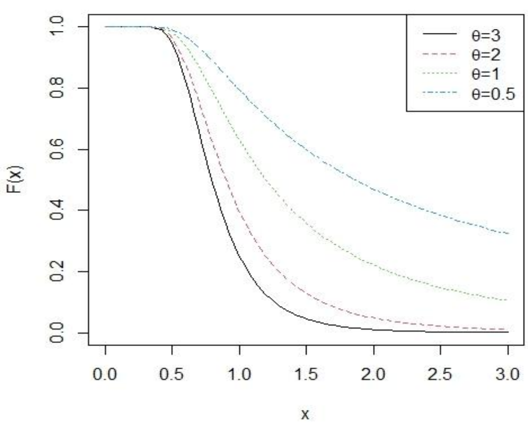

Fig. 1. Plot of cumulative density function at different value of parameter

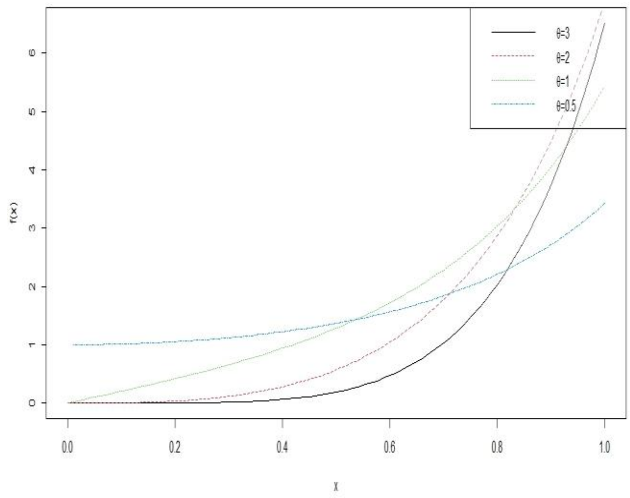

Fig. 2. Plot of probability density function at different value of parameter

Types of loss functions: -

If $\hat{\theta}$ represent of estimator for the shape parameter , then for:

1. Squared error loss function (SELF): SELF defined as:(2011)

$$L(\hat{\theta},\theta) = c(\hat{\theta}-\theta)^2 \;;\; \hat{\theta}_{self} = E(\theta) \tag{7}$$

2. Linear exponential loss function: the loss function defined as: (2011)

$$L(\hat{\theta},\theta) = \left(e^{c(\hat{\theta}-\theta)} - c(\hat{\theta}-\theta) - 1\right);\; \hat{\theta}_{linex} = -\frac{1}{c} lnE\left(e^{-c\theta}\right) \tag{8}$$

3. Precautionary loss function: the Precautionary loss function defined as: (2012)

$$L(\hat{\theta},\theta) = \frac{(\hat{\theta}-\theta)^2}{\hat{\theta}} \;;\; \hat{\theta}_{plf} = \sqrt{E(\theta^2)} \tag{9}$$

4. Entropy loss function (ELF): ELF defined as: (2011)

$$L(\hat{\theta},\theta) = \left(\frac{\hat{\theta}}{\theta}\right)^t - t\ln\left(\frac{\hat{\theta}}{\theta}\right)^t - 1;\; \hat{\theta}_{elf} = \left(E(\theta^{-t})\right)^{-\frac{1}{t}} \tag{10}$$

5. De Groot loss function (DLF): DLF defined as: (2011)

$$L(\hat{\theta},\theta) = \left(\frac{(\hat{\theta}-\theta)}{\hat{\theta}}\right)^2;\; \hat{\theta}_{Dlf} = \frac{E(\theta^2)}{E(\theta)} \tag{11}$$

6. Non- Linear exponential loss functions is defined as:

$$L(\hat{\theta},\theta) = \left(e^{c(\hat{\theta}-\theta)} + c\,(\hat{\theta}-\theta)^2 - c(\hat{\theta}-\theta) - 1\right);\; \hat{\theta}_{Nlinex} = \frac{1}{c+1} lnE\left(e^{-c\theta}\right) - 2E(\theta) \tag{12}$$

The Posterior distributions with different priors

For the random variable X, the posterior density function of the shape parameter θ is:

$$P(\theta|\underline{x}) = \frac{L(\theta|\underline{x}).p(\theta)}{\int_0^\infty L(\theta|\underline{x}).p(\theta)d\theta} \tag{13}$$

For “Bayesian estimation”, we describe two distinct posterior distributions under complete samples, and two distinct prior distributions for the shape parameter.

The Jeffery’s prior, for the parameter $(\boldsymbol{\theta})$ is:

$$p(\theta) = \frac{1}{\theta};\ \theta > 0 \tag{14}$$

Then under the assumption of this prior distribution and by equation (13), the posterior distribution will be as:

$$p_j(\theta|\underline{x}) = \frac{q_1 \theta^n e^{-\theta[\sum_{i=1}^{n} \ln(1-e^{-x_i^2})^{-1}]} \frac{1}{\theta}}{\int_0^\infty q_1 \theta^n e^{-\theta[\sum_{i=1}^{n} \ln(1-e^{-x_i^2})^{-1}]} \frac{1}{\theta} d\theta} \tag{15}$$

$$p_j(\theta|\underline{x}) = \frac{\left(\sum_{i=1}^{n} \ln(1-e^{-x_i^2})\right)^n}{\Gamma(n)} \theta^{n-1} e^{-\theta\left(\sum_{i=1}^{n} \ln(1-e^{-x_i^2}\right)^{-1}} \qquad \theta > 0 \tag{16}$$

The Gamma prior, considered to be:

$$p(\theta) = \frac{b^a \theta^{a-1}}{\Gamma(\alpha)} e^{-\theta b}, \theta > 0,\ a, b > 0 \tag{17}$$

By equation (13) the posterior distribution under the assumption Gamma prior is:

$$p_j(\theta|\underline{x}) = \frac{q_1 \theta^n e^{-\theta[\sum_{i=1}^{n} \ln(1-e^{-x_i^2})^{-1}]} \frac{b^a \theta^{a-1}}{\Gamma(\alpha)} e^{-\theta b}}{\int_0^\infty q_1 \theta^n e^{-\theta[\sum_{i=1}^{n} \ln(1-e^{-x_i^2})^{-1}]} \frac{b^a \theta^{a-1}}{\Gamma(\alpha)} e^{-\theta b} d\theta} \tag{18}$$

$$p_j(\theta|\underline{x}) = \frac{\left(\sum_{i=1}^{n} \ln(1-e^{-x_i^2})^{-1} + b\right)^{n+a}}{\Gamma(n+a)} \theta^{n+a-1} e^{-\theta[\sum_{i=1}^{n} \ln\left(1-e^{-x_i^2}\right)^{-1} + b]};\quad \theta > 0 \tag{19}$$

Bayesian Estimators under Jeffrey Prior using the six different loss functions:

$$\hat{\theta}_{SELF} = \frac{n}{\sum_{i=1}^{n} \ln(1-e^{-x_i^2})^{-1}} \tag{20}$$

$$\hat{\theta}_{linex} = \frac{1}{c} \ln\left(\frac{\sum_{i=1}^{n} \ln(1-e^{-x_i^2})^{-1}}{\ln(1-e^{-x_i^2})^{-1} + c}\right) \tag{21}$$

$$\hat{\theta}_{plf} = \sqrt{\frac{n(n+1)}{\left(\ln(1-e^{-x_i^2})^{-1}\right)^2}} \tag{22}$$

$$\hat{\theta}_{elf} = \frac{\frac{\Gamma(n)}{\Gamma(n-t)}^{1/t}}{\sum_{i=1}^{n} \ln(1-e^{-x_i^2})^{-1}}$$
(23)

$$\hat{\theta}_{DLF} = \frac{(n+1)}{\sum_{i=1}^{n} \ln(1-e^{-x_i^2})^{-1}} \tag{24}$$

$$\hat{\theta}_{NLinex} = \frac{\left(\ln\left(\frac{\sum_{i=1}^{n} \ln\left(1-e^{-x_i^2}\right)^{-1}}{\sum_{i=1}^{n} \ln\left(1-e^{-x_i^2}\right)^{-1} + c}\right)^n - 2\frac{n}{\sum_{i=1}^{n} \ln\left(1-e^{-x_i^2}\right)^{-1}}\right)}{c+2} \tag{25}$$

Bayesian Estimators under Gamma Prior using the six different loss functions:

$$\hat{\theta}_{SELF} = \frac{n+a}{\sum_{i=1}^{n} \ln(1-e^{-x_i^2})^{-1}+b} \tag{26}$$

$$\hat{\theta}_{linex} = \frac{1}{c}\ln\left(\frac{\sum_{i=1}^{n} \ln(1-e^{-x_i^2})^{-1}+b}{\ln(1-e^{-x_i^2})^{-1}+c+b}\right)^{(n+a)} \tag{27}$$

$$\hat{\theta}_{plf} = \sqrt{\frac{(n+a)(n+a+1)}{\left(\ln(1-e^{-x_i^2})^{-1}+b\right)^2}}$$

$$\hat{\theta}_{elf} = \frac{\frac{\Gamma(n+a)}{\Gamma(n+a-t)}^{1/t}}{\sum_{i=1}^{n} \ln(1-e^{-x_i^2})^{-1}+b} \tag{28}$$

$$\hat{\theta}_{DLF} = \frac{(n++a+1)}{\sum_{i=1}^{n} \ln(1-e^{-x_i^2})^{-1}+b} \tag{29}$$

$$\hat{\theta}_{NLinex} = \frac{\left(\ln\left(\frac{\sum_{i=1}^{n} \ln\left(1-e^{-x_i^2}\right)^{-1}+b}{\sum_{i=1}^{n} \ln\left(1-e^{-x_i^2}\right)^{-1}+b+c}\right)^{n+a} - 2\,\frac{n}{\sum_{i=1}^{n} \ln\left(1-e^{-x_i^2}\right)^{-1}+b}\right)}{c+2} \tag{30}$$

3. SIMULATION STUDY

In this section we mainly perform some simulation experiments to observe the behaviour of the different Bayes estimators for the shape parameter proposed in the previous section (3), using different sample sizes “n= 10, 20, 30, 50, 75 and 100”and for two distinct parameters values “θ= 1.5 and 3”, by applying the Monte Carlo simulation to compare the performance of these estimators using the mean squared error (MSE). All results are computed by R program based on replications and for two different put “a = 3, b= 0.8, c= 1; a = 4, b= 1.2, c= 1”.

The mean and MSE values for the Bayesian estimator, are recorded in tables (1), (2), (3) and (4).

Table 1. The mean value of $\hat{\boldsymbol{\theta}}$, when $\boldsymbol{\theta} = \mathbf{1.5}$

n	Jeffery prior					
	B$_{SELF}$	**B$_{PLF}$**	**B$_{DLF}$**	**B$_{LINEX}$**	**B$_{ELF}$**	**B$_{NLINEX}$**
10	1.6566	1.7356	1.8124	1.5118	1.4047	1.6125
20	1.5573	1.5860	1.6253	1.4871	1.4301	1.5464
30	1.5502	1.5657	1.6017	1.5105	1.4622	1.5345
50	1.5264	1.5312	1.5462	1.5022	1.4713	1.5144
75	1.5219	1.5217	1.5320	1.5043	1.4811	1.5132
100	1.5191	1.5262	1.5245	1.5032	1.4761	1.5112

n	Gamma prior					
	B$_{SELF}$	**B$_{PLF}$**	**B$_{DLF}$**	**B$_{LINEX}$**	**B$_{ELF}$**	**B$_{NLINEX}$**
10	1.8812	1.9512	2.0247	1.7456	1.6615	1.8357
20	1.6832	1.7274	1.7453	1.6231	1.5634	1.6606
30	1.6452	1.6597	1.67082	1.5843	1.5407	1.6328
50	1.5782	1.5832	1.6182	1.5523	1.5376	1.5632
75	1.5472	1.5638	1.5679	1.5413	1.5246	1.5539
100	1.5452	1.5532	1.5509	1.5234	1.5232	1.5423

n	Gamma prior					
	B_{SELF}	B_{PLF}	B_{DLF}	B_{LINEX}	B_{ELF}	B_{NLINEX}
10	1.9054	1.6345	2.1420	1.7657	1.7032	1.8632
20	1.7034	1.5604	1.7676	1.6345	1.6245	1.7682
30	1.6401	1.5456	1.6875	1.6208	1.5934	1.6456
50	1.5789	1.5345	1.6556	1.5753	1.5443	1.5965
75	1.5556	1.5245	1.5236	1.5532	1.5265	1.5534
100	1.5434	1.5156	1.5123	1.5256	1.5134	1.5431

Table 2. The mean value of $\hat{\theta}$, when $\theta = 3$

n	Jeffery prior					
	B_{SELF}	B_{PLF}	B_{DLF}	B_{LINEX}	B_{ELF}	B_{NLINEX}
10	3.3243	3.4652	3.5247	2.8184	2.8125	3.1517
20	3.1145	3.1924	3.3458	2.7857	2.7823	3.1386
30	3.0764	3.1778	3.2764	2.6219	2.7225	3.0261
50	3.0621	3.1231	3.1289	2.6708	2.7706	3.0226
75	3.0423	3.0645	3.0643	2.5823	2.5669	3.0227
100	3.0351	3.0545	3.0565	2.4596	2.5636	3.0200

n	Gamma prior (a= 3, b = 0.8)					
	B_{SELF}	B_{PLF}	B_{DLF}	B_{LINEX}	B_{ELF}	B_{NLINEX}
10	3.3456	2.8732	3.4729	2.9598	2.9556	3.2162
20	3.2593	2.8978	3.3543	2.9372	2.9618	3.1032
30	3.1208	2.9343	3.2467	2.9784	2.9738	3.0432
50	3.0878	2.9991	3.1343	3.0023	3.0056	3.0634
75	3.0695	2.9922	3.0881	3.0092	3.0084	3.0457
100	3.0414	2.9821	3.0652	3.0076	3.0073	3.0465

n	Gamma prior (a= 4, b = 1.2)					
	B_{SELF}	B_{PLF}	B_{DLF}	B_{LINEX}	B_{ELF}	B_{NLINEX}
10	2.9675	2.9013	2.9292	2.9598	2.9556	3.2162
20	3.1278	2.8323	3.2567	2.9372	2.9618	3.1032
30	3.1149	2.9324	3.2076	2.9784	2.9738	3.0432
50	3.0643	2.9435	3.1465	3.0023	3.0056	3.0634
75	3.0351	2.9772	3.0881	3.0923	3.0084	3.0457
100	3.0484	2.9822	3.0652	3.0772	3.0073	3.0465

Table 3. The MSE value of $\hat{\theta}$, when $\theta = 1.5$

n	Jeffry's prior					
	B_{SELF}	B_{PLF}	B_{DLF}	B_{LINEX}	B_{ELF}	B_{NLINEX}
10	0.3343	0.2302	0.4765	0.2187	0.2343	0.2934
20	0.1345	0.1153	0.1627	0.10834	0.1134	0.1243
30	0.0807	0.0786	0.1041	0.0797	0.0792	0.0831
50	0.0486	0.5674	0.0534	0.0468	0.0245	0.0533
75	0.0323	0.3493	0.3391	0.0310	0.0302	0.0332
100	0.0241	0.2321	0.2383	0.2354	0.0232	0.0245
n	**Gamma prior (a= 4, b = 1.2)**					
	B_{SELF}	B_{PLF}	B_{DLF}	B_{LINEX}	B_{ELF}	B_{NLINEX}
10	0.4323	0.2141	0.6014	0.2808	0.2543	0.3754
20	0.1621	0.1054	0.2062	0.1243	0.1162	0.1489
30	0.1204	0.0802	0.1323	0.0932	0.0853	0.1032
50	0.0573	0.0453	0.0637	0.0549	0.0483	0.0549
75	0.0352	0.0304	0.0365	0.0343	0.0323	0.0421
100	0.0244	0.0234	0.0275	0.0256	0.0231	0.0245

n	Gamma prior (a= 4, b = 1.2)					
	B_{SELF}	B_{PLF}	B_{DLF}	B_{LINEX}	B_{ELF}	B_{NLINEX}
10	0.4327	0.2145	0.6013	0.2918	0.2532	0.3767
20	0.1623	0.1052	0.2067	0.1242	0.1160	0.1476
30	0.1101	0.0801	0.1322	0.0912	0.0857	0.1034
50	0.0574	0.0463	0.0651	0.0507	0.0478	0.054I
75	0.0351	0.0306	0.0386	0.0334	0.0312	0.0321
100	0.0251	0.0224	0.0275	0.0297	0.0251	0.0242

Table 4. The MSE value of $\hat{\theta}$, when $\theta = 3$

n	Jeffry's prior					
	B_{SELF}	B_{PLF}	B_{DLF}	B_{LINEX}	B_{ELF}	B_{NLINEX}
10	1.3456	1.6709	1.9305	0.6836	0.9347	1.0532
20	0.5326	0.5867	0.6467	0.3875	0.4587	0.4357
30	0.3189	0.3343	0.3568	0.2557	0.2861	0.2856
50	0.2018	0.2096	0.2204	0.1743	0.1865	0.1904
75	0.1306	0.1388	0.1395	0.1183	0.1228	0.1256
100	0.0972	0.0976	0.0971	0.0876	0.0912	0.0912
n	**Gamma prior (a= 3, b = 0.8)**					
	B_{SELF}	B_{PLF}	B_{DLF}	B_{LINEX}	B_{ELF}	B_{NLINEX}
10	0.8503	1.0097	1.2108	0.4493	0.5734	0.6532
20	0.4245	0.4609	0.5213	0.2956	0.3453	0.3657
30	0.2867	0.3078	0.3284	0.2205	0.2423	0.2556
50	0.1987	0.1997	0.2121	0.1612	0.1715	0.1784
75	0.1346	0.1308	0.1353	0.1122	0.1176	0.1216
100	0.0932	0.0961	0.0976	0.0845	0.0861	0.0812

According to Tables (3) and (4), as sample size rises, the rate at which estimates approach the true value of the shape parameter increases. Based on the best estimates from experiments 3 and 4, it can be concluded that the performance of the Bayes estimator with the Gamma prior and the linear exponential loss function is preferable to that of other priors with other priors and for various sample sizes.

3.1 Data Analysis

This section is devoted to illustrate the practical applications of the proposed exponentiated Rayleigh distribution. In order to assess the flexibility of the new model, we analyze one real life data sets taken from literature and the numerical results of exponentiated Rayleigh distribution are compared with its sub-models, namely Rayleigh distribution (RD), Weibull distribution (WD), exponential distribution (ED) and generalized exponential distribution (GED). The model selection is carried out by using different model selection criterions including the negative log-likelihood, Akaike information criteria (AIC) (Akaike 1974), Schwarz Information Criteria (SIC) (Schwarz 1978), Corrected Akaike information criteria (AICC) (Bazdogan 1987). Also, Kolmogorov-Simonov test statistics along with corresponding p-value has been calculated.

Data set 1: The data set due to Smith and Naylor (1987) consists of 63 observations of the strengths of 1.5 cm glass fibres, originally obtained by workers at the UK National Physical Laboratory. This data set was also analysed by Oguntunde et al. (2015) to demonstrate the applicability of Weibull-Exponential distribution.

The data are: "0.55, 0.74, 0.77, 0.81, 0.84, 0.93, 1.04, 1.11, 1.13, 1.24, 1.25, 1.27, 1.28, 1.29, 1.30, 1.36, 1.39,1.42, 1.48, 1.48, 1.49, 1.49, 1.50, 1.50, 1.51, 1.52, 1.53, 1.54, 1.55, 1.55, 1.58, 1.59, 1.60, 1.61, 1.61,1.61, 1.61, 1.62, 1.62, 1.63, 1.64, 1.66, 1.66, 1.66, 1.67,1.68, 1.68, 1.69, 1.70, 1.70, 1.73, 1.76, 1.76,1.77, 1.78, 1.81, 1.82, 1.84, 1.84, 1.89, 2.00, 2.01, 2.24".

Table 5. ML estimate and the statistic -2l, AIC, SIC, AICC using data set 1

Model	ML estimates	-2l	AIC	SIC	AICC
ERD	1.4997	172.22	178.43	185.23	179.49
RD	1.635	180.35	184.23	189.35	185.06
WD	2.543	196.23	198.67	200.50	198.58
GED	5.783	194.67	202.34	213.67	185.76

4. CONCLUSIONS

The methods described to build a full framework to accommodate academic research and engineering applications seeking to implement modern computational based classical as well as Bayesian approaches related to exponentiated Rayleigh distribution, especially in the area of reliability. We have proposed an integrated procedure for Bayesian inference using Markov chain Monte Carlo methods. For the sake of comparison, we have discussed the maximum likelihood estimation. The above study makes the suggestion that when =3, the performance of the Bayes estimator under the Gamma prior with linear loss function, records full appearance "for all sample sizes," as the best prior distribution, and utilising various loss functions for the entire data set.

A simulation study is carried out to investigate the behaviour of ML estimates for finite sample size. The estimation of parameters is approached by the method of maximum likelihood estimation. The applications of the power exponentiated Rayleigh distribution to real data are provided which show that the new distribution can be used quite effectively to provide better fits than the other competing distributions. We prospect that the proposed model will draw wider applications in statistics

COMPETING INTERESTS

Authors have declared that no competing interests exist.

REFERENCES

1. Burr IW. Cumulative frequency distribution. Annals of Mathematical Statistics. 1942;13:215-232.
2. Surles JG, Padgett WJ. Inference for reliability and stress -strength for a scale Burr Type X distribution. Lifetime Data Analysis. 2001;7:187-200.
3. Sartawi HA, Abu-Salih MS. Bayes prediction bounds for the Burr Type X model. Comm., Statist. Theory Methods. 1991;20:2307-2330.
4. Jaheen ZF. Bayesian approach to prediction with outliers from the Burr Type X model. Microelectronic Reliability. 1995;35:45-47.
5. Ahmad KE, Fakhry, Jaheen ZF. Empirical Bayes estimation of p(y<x) and characterization of Burr Type X model. Journal of Statistical Planning and Inference. 1997;46:297- 308.
6. Karam NS, Jbur AK. "Bayesian Analyses of the Burr Type Distribution under Doubly Type censored samples using different Priors and Loss functions" Mathematical; 2014.
7. Feroze N, Aslam M. Bayesian Analysis of Burr Type X distribution under Complete and Censored Samples. International Journal of Pure and Applied Sciences and Technology. 2012;11(2):16-28.
8. Sindhu TN, Feroze N, Aslam M. Bayesian Analysis of the Kumaraswamy Distribution under Failure Censoring Sampling Scheme. International Journal of Advanced Science and Technology. 2013;51:39-58.
9. Jonson NL, Kotz S, Balakrishnan N. Continuous univariate distribution. Volume 1, Second Edition, John and Wiley Sons, Inc., New York; 1994.
10. Voda VG. A new generalization of Rayleigh distribution. Reliab: Theory Appl. 2007;2(2):47-56.
11. Raqab MZ, Madi MT. Bayesian analysis for the exponentiated Rayleigh distribution. Int J Stat. 2009;LXVII: 269-288
12. Merovci F. Transmuted Rayleigh distribution. Austrian J Stat. 2013;42(1):21-31.
13. Dey S, Dey T, Kundu D. Two-parameter Rayleigh distribution: Different methods of estimation. Am.J Math Manag Sci. 2014;33:55-74
14. Ahmad A, Ahmad SP, Ahmed A. Characterization and estimation of transmuted Rayleigh distribution.J Stat Appl Prob. 2015;4(2):315-321.
15. Saima N, Ahmad SP, Ahmed A. Bayesian analysis for generalized Rayleigh Distribution. J MathTheory Model. 2016;6(5):2224-580.

16. Ateeq K, Qasim BT, Alvi RA. An extension of Rayleigh distribution and applications. Cogent Math Stat. 2019;6(1):1-16.
17. Islam S. Loss functions, Utility functions and Bayesian Sample Size Determination. PH.D. Thesis, University of London; 2011.
18. Singh SK, Singh U, Kumar D. Bayesian Estimation of the Exponentiated Gamma Parameter and Reliability function under asymmetric loss function. Statistical Journal. 2011;9(3):247–260.
19. Principles. International Journal of Mathematical, Computational, Physical and Quantum Engineering. 2014; 8(1).
20. Sindhu TN, Aslam M. On the Parameter of the Burr Type X under Bayesian Theory and Modeling. 2014;4(14):9-18.

Peer-review history:
The peer review history for this paper can be accessed here:
https://www.sdiarticle5.com/review-history/95476