British Journal of Mathematics & Computer Science
6(6): 475-480, 2015, Article no.BJMCS.2015.094
ISSN: 2231-0851

SCIENCEDOMAIN *international*
www.sciencedomain.org

On Modelling of Promiscuous Lifestyle; An SIR Dynamics

W. Obeng-Denteh[1], **B. A. Obeng**[1*] **and G. O. Fosu**[2]

[1] *Department of Mathematics, Kwame Nkrumah University of Science and Technology, Kumasi, Ghana.*
[2] *Department of ICT and Mathematics, Presbyterian University College, Abetifi, Ghana.*

Article Information
DOI: 10.9734/BJMCS/2015/11815
Editor(s):
(1) Jaime Rangel-Mondragon, Faculty of Informatics, Queretaros Institute of Technology, Mexico and Faculty of Computer Science, Autonomous University of Quertaro, Mexico.
(2) Paul Bracken, Department of Mathematics, The University of Texas-Pan American Edinburg, TX 78539, USA.
Reviewers:
(1) Isack Kibona, Department of Science and Business Management, Mbeya University of Science and Technology, Tanzania.
(2) Anonymous, P.R. China.
(3) Gbolahan Bolarin, Mathematics and Statistics Department, Federal University of Technology, Minna, Niger State, Nigeria.
(4) Anonymous, China.
(5) Anonymous, P.R. China.
Complete Peer review History:
http://www.sciencedomain.org/review-history.php?iid=736&id=6&aid=7823

Original Research Article

Received: 05 June 2014
Accepted: 12 August 2014
Published: 20 January 2015

Abstract

An epidemiological model depicting the dynamics of campus sex among students is analyzed in this study. For the purpose of this research, sexual activities among students is considered as a disease called SESEX. The Susceptible - Infected - Recovered (SIR) model is used in analyzing the dynamics in this scenario. Two equilibrium points were found, a disease-free equilibrium and an endemic equilibrium point. The model suggests that, admitting new students without the infection plays a significant role in the reduction of SESEX on campus. Thus, minimizing recruitment of infected students reduces the spread of the disease.

Keywords: Students, sexual practice, SIR model.

2010 Mathematics Subject Classification: 53C25; 83C05; 57N16

**Corresponding author: E-mail: obenadu@yahoo.com*

1 Introduction

The past decades has witnessed a series of research works on the addiction and spread of disease in many areas of studies due to its importance ([1], [2], [3], [4], [5], [6], [7]). Modelling has become a predictive tool which helps to determine the existence and treatment strategies of certain diseases. It has become the need to develop useful models which will give practitioners and academician valuable predictions. Mathematical models for disease spread and addiction subjects, or more generally epidemics, are typically based on differential equations with an in-built threshold that determine the behaviour of the system. The importance of the threshold is to determine the existence of an epidemic based on the parameter values when exceeding the threshold. The epidemic may include, drug taken, alcoholism, obesity, prostitution and many others.

The spread of any disease on campus has both short and long term negative consequence on campus environment, thus: students, lecturers, other workers, academic works and public health. In this study we categorize student-sex as a disease on campuses. The term SESEX is considered as engaging in a sexual act when not married for any reason or having sexual act outside your marriage partner.

Studies on student-sex has been carried out by many researchers in many fields of studies ([8], [9], [10], [11], [12]). To the best of the researchers knowledge, there has not been any work to compartmentalize students sexual activities on campus. This research is to analyse premarital and extramarital sexual activities among students on campus using mathematical model. The model depict the existence and spread of SESEX as an epidemic disease on campuses. An epidemiological model capturing the dynamics of SESEX on campuses is illustrated with an SIR model. The model together with the scenario in question is a hypothetical case but not real issue based on actual data. Actual data could be used to test the efficacy of the model which is undoubted.

2 The Model

The model captures the dynamics of SESEX and it spread on campuses. It is a prototype of the usual SIR model. The total population (total students) is divided into three compartments; the non infected student, infected students and the recovered group.

Non infected students (S) are the susceptible group. They are the students admitted into the school population without the SESEX-disease. Infected students (P) are those whose sexual habit and associated behaviour have negative consequence on other students. Recovered students (R) are those who have stopped illegal sex for a minimum of 12 months.

An individual joins campus as a susceptible or an infected student, and leave in any of the three state. It is considered that, students admitted are automatically 'at risk' of becoming infected. This infectious rate is given as $\alpha(S + P + R)$. The student therefore exits the susceptible, infected and recovered class naturally at rates αS, $(\alpha + \sigma)P$ and αR respectively. Where αN is the number of susceptible students admitted into the system every academic year; α is the entering and departure rate from campus environment; β is the transmission rate of the disease; γ is the rate of recovery from SESEX; κ is the relapse rate from recovered class to the infected compartment; and σ is the rate at which the infected population are recruited fresh into campus. The model refers to the assumption that students within each compartment are similar with regards to their behaviours. The transition to a lower state is assumed to be a recovery.

Assumptions of the Model

The following are the assumptions of the model:

- Constant population size.

- Non infected includes never and recovered admitted.
- Recovered can only relapse.
- The rate at which the disease is acquired is proportional to the product of susceptible and infective present.
- Transmission is between students only.
- The effects of breaks and vacations are not considered.

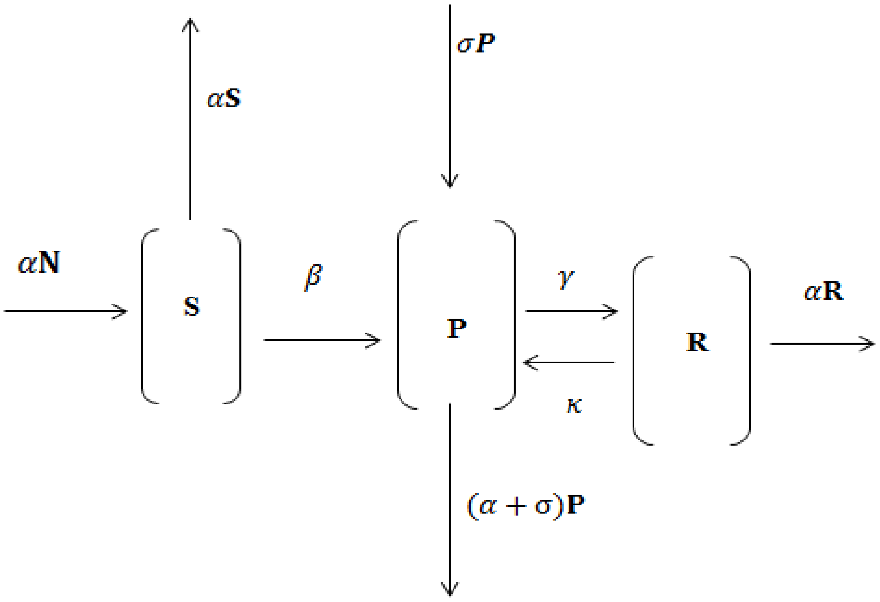

Figure 1: Compartmental diagram

The equations from the compartmental diagram (see figure 1) are:

$$\begin{aligned}\frac{ds}{dt} &= \alpha - \beta sp - \alpha s\\ \frac{dp}{dt} &= \sigma p + \beta sp + \kappa pr - \gamma p - (\alpha+\sigma)p\\ \frac{dr}{dt} &= \gamma p - \kappa pr - \alpha r\end{aligned} \tag{2.1}$$

where $s + p + r = 1$. These are modeled by rescaling the sub population groups.

3 Model Analysis

At the steady state, we obtained: $(s^*, p^*, r^*) = (1, 0, 0)$ which is the disease free equilibrium. We further determined whether SESEX will persist or not. The Jacobian matrix obtained from the differential equations is given as:

$$J(s,p,r) = \begin{bmatrix} -\beta p - \alpha & -\beta s & 0 \\ \beta p & \beta s + \kappa r - \gamma - \alpha & \kappa p \\ 0 & \gamma - \kappa r & -\kappa p - \alpha \end{bmatrix}$$

The respective lambda values below are the results of the eigenvalues.

$$\lambda_1 = -\alpha, \lambda_2 = -\gamma \text{ and } \lambda_3 = \beta - \alpha - \gamma$$

The basic reproductive number R_0 was calculated using Heffernan approach [13]. Hence, $R_0 = \frac{\beta}{\alpha + \gamma - \beta}$. For an unstable disease-free equilibrium, $\beta > \frac{\alpha + \gamma}{2}$ and $\beta < \frac{\alpha + \gamma}{2}$ for a stable case. From the R_0 obtained, the transmission rate β relative to the recovery rate γ and departure rate α has a role in determining whether or not the SESEX exist on campuses.

Other equilibrium values obtained at the steady state were $(s^*, p^*, r^*) = (x, y, z)$. Where,

$$x = \frac{\alpha}{\alpha + \beta p}$$
$$y = \frac{\sqrt{\beta^2(\gamma^2 + \kappa^2 + \alpha^2 + 2\alpha\gamma + 2\alpha\kappa - 2\gamma\kappa) + \kappa^2(\alpha^2 - 2\alpha\beta) - 2\alpha\gamma\beta\kappa - 2\alpha^2\beta\kappa}}{2\beta\kappa}$$
$$z = \frac{\gamma p}{\alpha + \kappa p}$$

Solving for the eigenvalues,$\lambda_{1,2,3}$, we obtained the characteristic equation: $\lambda^3 - x\lambda^2 - y\lambda + z = 0$. Where

$x = (-\beta p - \alpha + \beta s + \kappa r - \gamma - \alpha - \kappa p - \alpha)$

$y = -(\beta s)(\beta p) - (-\beta p - \alpha)(\beta s + \kappa r - \gamma - \alpha) - (-\beta p - \alpha)(-\kappa p - \alpha) - (\beta s + \kappa r - \gamma - \alpha)(-\kappa p - \alpha) - (\kappa p)(\gamma - \kappa r)$

$z = (-\beta p - \alpha)(\beta s + \kappa r - \alpha - \gamma)(-\kappa p - \alpha) - (-\beta p - \alpha)(\kappa p)(\gamma - \kappa r) - (-\beta s)(\beta p)(-\kappa p - \alpha)D$

$\lambda_1 = -z$ and $\lambda^2 - T\lambda + D = 0$, where T is the trace and D the determinant. $\lambda_{2,3} = \frac{T \pm \sqrt{T^2 - 4D}}{2}$

If $T < 0, D > 0, \Rightarrow T^2 - 4D < 0$. Hence λ_2, λ_3 have negative real parts by the Trace-Determinant plane. The system is asymptotically stable at this endemic state, indicating that the disease will exist among students.

4 Numerical Analysis and Results

A set of numerical values were randomly chosen for the various parameters to perform the sensitivity analysis of the this model: $\alpha = 0.38, \beta = 0.67, \gamma = 0.54, \sigma = 0.123, \kappa = 0.45$.

R_0 was calculated using $R_0 = \frac{\beta}{\alpha + \gamma - \beta}$ and yields $R_0 = \frac{0.67}{0.38 + 0.54 - 0.67} = 2.68$

Since R_0 is greater than one, the disease free equilibrium is asymptotically unstable. Thus, the disease-SESEX does not die out at this point.

We varied the model input parameters to determine its impact on the reproductive number. It was realized that R_0 is more sensitive to infectious rate β as compared to the recovery rate γ, and departure rate α . As the rate of β increases, the reproductive number also increases, showing the spread of SESEX on campus. Also, the reproductive number decreases when recovery rate γ, or departure rate α increases.

This is illustrated in figure 2.

It was also observed that, as the infectious rate increases, the susceptible and the recovery class decreases with time. But the decrease in the recovery is not as much as that of the susceptible, indicating a net increase of infected students. Since the rate of recovery is relatively slow, there is the existence and increase in the number of infected student with time.

From

$$J_s = \begin{bmatrix} -\beta p - \alpha & -\beta s & 0 \\ \beta p & \beta s + \kappa r - \gamma - \alpha & \kappa p \\ 0 & \gamma - \kappa r & -\kappa p - \alpha \end{bmatrix} = \begin{bmatrix} -0.6744 & -0.3775 & 0 \\ 0.2944 & -0.0881 & 0.2329 \\ 0 & 0.0856 & -0.6129 \end{bmatrix}$$

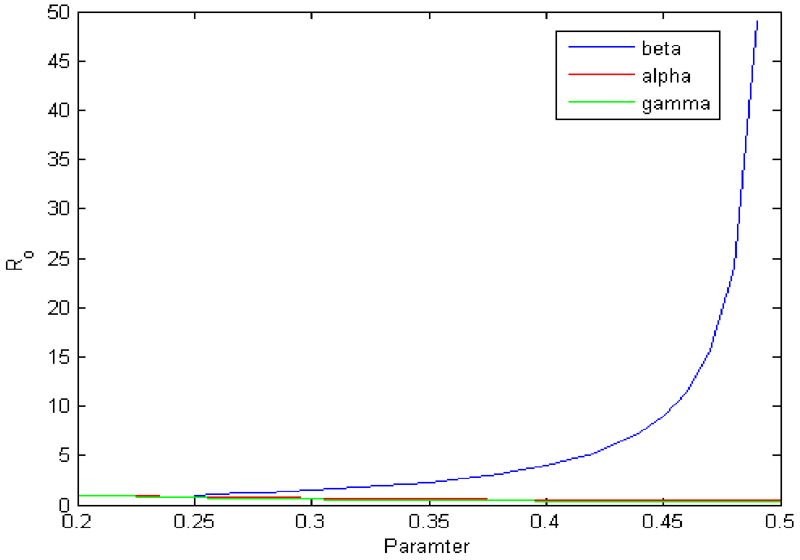

Figure 2: Graph of R_0 for students against changes in Parameters

The eigenvalues are: $\lambda_1 = -0.3800, \lambda_2 = -0.4083$ and $\lambda_3 = -0.5872$. By the eigenvalues, the system is asymptotically stable, and the disease-prostitution will persist among students.

5 Conclusion

In this work, sexual practice by students on campus were analysed with a set of ODEs. A prototype SIR model is used to analysis the dynamics of SESEX on campus. The steady state behavior of the system was performed with chosen set of values. The disease-free equilibrium was asymptotically unstable. It was realized that, the eigenvalues of the endemic Jacobian matrix was stable. By the graphical representation, the model was more sensitive to the transmission rate as compared to the recovery and departure rate. The numerical results showed unstable disease free equilibrium point and stable endemic equilibrium point, implying SESEX cannot die out on campuses but its spread and infection could be minimized.

Competing Interests

The authors declare that no competing interests exist.

References

[1] Andersson M. SIR epidemic on a configuration model network Mathematical Statistics Stockholm University; 2013.

[2] Antonio M. Correia, Mena FC, Soares AJ. An application of the SIR model to epidemics in Portugal; 2011.

[3] Bhunu CP. A mathematical analysis of alcoholism *World Journal of Modelling and Simulation.* 2012;8:124-134.

[4] Busenberg S, Cooke K, Hsieh Y. A Model for HIV in Asia. Elsevier; 1994.

[5] Diekmann O, Heesterbeck JAP. Mathematical epidemiology of infectious diseases. Wiley, New York; 2000.

[6] Hung G, Takeuchi Y. Global analysis on delay epidemiological dynamic models with nonlinear incidence. Journal of mathematical Biology. 2011;63:125-139.

[7] Kermack WO, McKendrick AG. Contributions to the mathematical theory of epidemics. Proc. Roy. Soc. Ser A. 1927;115:700-721.

[8] Hardit S. Predicting sexual aggression among college men: The role of male peer groups and sexualized media. PhD thesisUniversity of Illinois at Urbana-Champaign, Urbana, Illinois; 2012.

[9] Kusumastuti I. Association between knowledge, attitudes and sexual practices among unmarried Indonesian young adults: A study from Indonesian Young adults reproductive health survey (Iyarhs). 4th International Conference on Reproductive Health and Social Sciences Research. 2007;273:93-104.

[10] Sachdev P. Sex on campus: A preliminary study of knowledge,attitude and behaviour of university students in Delhi, India. Journal of Biosocial Science. 2000;30:95-105.

[11] Setyatama IP, Rimawati E, Shaluhiyah Z. The phenomenom of student prostitutes 'campus Chicken' in some university in Semarang. 4th International Conference on Reproductive Health and Social Sciences Research. 2007;273:239-245.

[12] Tura G, Alemseged F, Dejene S. Risky sexual behaviour and predisposing factors among students of Jimma university, Ethiopia. Ethiop J Health Sci. 2012;22(3).

[13] Heffernan JM, Smith RJ, Wahl LM. Perspectives on the basic reproduction ratio. Journal of the Royal Society Interface. 2005;2:281-293.

Peer-review history:
The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)
www.sciencedomain.org/review-history.php?iid=736&id=6&aid=7823