Journal of Advances in Mathematics and Computer Science

Volume 39, Issue 4, Page 1-9, 2024; Article no.JAMCS.113446

ISSN: 2456-9968

(Past name: British Journal of Mathematics & Computer Science, Past ISSN: 2231-0851)

Mathematical Investigation of Option Pricing using Black-Scholes-Merton Partial Differential Equation with Transaction Cost

Calvince Fwaga [a*], Wilys O. Mukuna [a] and Levi Otanga Olwamba [a]

[a] *Department of Mathematics Actuarial and Physical Sciences, University of Kabianga, Kenya.*

Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

Article Information

DOI: 10.9734/JAMCS/2024/v39i41878

Open Peer Review History:
This journal follows the Advanced Open Peer Review policy. Identity of the Reviewers, Editor(s) and additional Reviewers, peer review comments, different versions of the manuscript, comments of the editors, etc are available here: https://www.sdiarticle5.com/review-history/113446

Received: 28/12/2023
Accepted: 04/03/2024
Published: 11/03/2024

Original Research Article

Abstract

Over the years studies have been done on option pricing valuation. The world market economies have experienced tremendous asset price fluctuations since 1980s. For this reason, efforts have been directed towards developing reliable and more accurate option pricing models. Black-Scholes-Merton model has so far been proved to be the most powerful and significant tool for the valuation of an option. However, its assumption of zero transaction cost on asset pricing yields inaccurate option values. The study investigates the effects of transaction cost on call and put option of an asset price using a two-dimensional Black-Scholes-Merton partial differential equation. The Dufort-Frankel Finite Difference Method is used to approximate the solution to the BSM model equation describing the value of an option with boundary conditions. The simulation is done with

**Corresponding author: E-mail: fwagacal91@gmail.com;*

the aid of MATLAB software program. The effects of incorporating transaction cost on the two assets prices on the value of an option using BSMPDE are determined. From the study, it is established that as transaction cost increases, the call and put option values decrease. The effects of incorporating transaction cost on the values of call and put option are shown in tabular form and graphically. These results are useful to the investors in computing possible returns on investment based on more accurate asset pricing and to the government on policy formulation in controlling prices in stock exchange market.

Keywords: Black-Scholes-Merton partial differential equation; option value; dufort-Frankel scheme; Transaction cost.

1 Introduction

Before the discovery of Black-Scholes model by Fischer Black, Myron Scholes and Robert Merton in 1973, there was no standard method of option pricing agreed upon by the option traders. Traders majorly relied on their intuition to price options. A breakthrough was later seen when Black-Scholes-Merton model became the most powerful and significant tool for the valuation of an option. Fluctuation in market prices of assets prompted rigorous mathematical and probabilistic concepts through the theory of stochastic process, also referred to as Wiener process [1]. The concepts solved the challenges in option valuation and gave new mathematical ideas that provided solutions to problems in finance and other fields. [2] developed a modified Black-Scholes-Merton model for Option Pricing. The model provided flexibilities for the markets. The study suggested that conformable Black-Scholes-Merton model may provide a way of valuing European call option compared to classical Black-Scholes model and fractional Black-Scholes model.

The study further showed that a more robust statistical procedure is required to improve the accuracy of option valuation. Additionally, the study suggested that an aspect of interest in asset pricing model to be considered is that of the transaction cost. [3] studied numerical solution of linear and non-linear Black-Scholes option pricing equations by means of semidiscretization technique. The study revealed that for a linear case, a fourth order discretization with respect to the underlying asset variables allows a better accurate approximation solution while for the non-linear case of interest modeling option pricing with transaction cost, semi discretization technique provides a competitive numerical solution. The study revealed that in practice, transaction costs arise when trading securities. The results of the study further demonstrated that although such transaction costs are generally small for institution investors, their influence lead to significant increase in the option price. [4] studied analytical solution of fractional Black-Scholes European option pricing equation by using Laplace transform. The study combined the form of Laplace and the homotopy perturbation method to obtain a quick and accurate solution to the fractional Black-Scholes equation with boundary condition for a European option pricing problem. The proposed scheme found the solution without any discretization or restrictive assumptions and free from round off errors thereby reducing numerical computation to a greater extent.[5] studied the Lie algebraic approach for determining pricing for trade account options. The research examined the options for the trade account using the Lie symmetry analysis. The study demonstrated that Lie symmetry technique can be used to analyze systemic problems in financial field although the method requires rigorous solutions to the numerous algebraic expressions. [6] carried out a study on the Walrasian-Samuelson Price Adjustment Model. In the study, an Ito method for modeling the changes of the market value of securities traded due to new information which affects the market asset supply and demand was introduced. It is formulated on the basis of; market supply, demand functions and the equilibrium price (Walrasian price) adjustment assumption, that the proportional price increase is driven by excess demand. The study established that if the supply and demand curves turned to be linear from the point of equilibrium, then the process changes to become logistic equation of Brownian motion with Wiener type of the random element. The study revealed that allowing transaction cost to be stochastic improves the accuracy of the option price prediction. This study forms a basis of our research on option pricing where the transaction cost is not zero. [7] carried out a study on simple formulas for pricing and hedging European options in the Finite Moment Log-stable model. The study showed that as stability parameters increase, both call and put option also increase. The study however assumed that volatility is constant and no transaction cost is incurred in trading. [8] applied a comparative analytical approach and numerical technique to determine the price of call option and put option of an underlying asset in the frontier markets so as to predict stock price. The study modified the Black-Scholes model so as to determine parameters such as strike price and expiration time. Machine learning approach was applied using Rapidminer software. The approach showed better results over classical Black-Scholes Option Pricing model. The study further considered numerical calculation of volatility and established that as the price of stocks goes up due to overpricing, volatility also increases at a high rate. The study did not however considered other parameters affecting option value such as transaction costs. [9] investigated numerical solutions of the non-linear Black-Scholes partial differential equation which often appears in financial markets for European option pricing in the presence of transaction costs. The study exploited the transformations for the computational purpose of a nonlinear Black-Scholes partial differential equation to modify as a nonlinear parabolic type of partial differential equation with initial and boundary conditions for both call and put options. The study derived several schemes using Finite Volume Method and Finite Difference Method. The study established that both methods provide numerical solutions which are closer to exact solutions.

The novelity of this study is to develop a mathematical model of Black-Scholes-Merton partial differential equation with transaction cost using Dufort-Frankel numerical scheme and show the effects of incorporating transaction cost on option value.

The rest of this paper is arranged in such a way that: section 2 presents the mathematical formulation of the problem, section 3 presents the results of the study and discussions, section 4 presents the conclusion and recommendation and section 5 presents suggestion for further studies.

2 Mathematical Formulation

Considering the two-dimensional Black-Scholes-Merton differential equation;

$$\frac{\partial V}{\partial t} + r\left[\frac{\partial V}{\partial S_1}S_1 + \frac{\partial V}{\partial S_2}S_2\right] + \frac{1}{2}\left[\sigma_1^2 S_1^2 \frac{\partial^2 V}{\partial S_1^2} + \sigma_2^2 S_2^2 \frac{\partial^2 V}{\partial S_2^2}\right] + \rho\sigma_1\sigma_2 S_1 S_2 \frac{\partial^2 V}{\partial S_1 \partial S_2} - rV = 0 \tag{1}$$

Assume the two assets S_1 and S_2 considers the transaction costs C_1 and C_2 meaning that at a time dt each of the assets incurs a transaction cost C_1S_1dt and C_2S_2dt respectively, V representing the volatility and r is the risk free rate. Therefore adding the transaction costs on each of the underlying assets from equation (1) becomes

$$\frac{\partial V}{\partial t} + r\left[\frac{\partial V}{\partial S_1}S_1 + \frac{\partial V}{\partial S_2}S_2\right] - \left[C_1S_1\frac{\partial V}{\partial S_1} + C_2S_2\frac{\partial V}{\partial S_2}\right] + \frac{1}{2}\left[\sigma_1^2 S_1^2 \frac{\partial^2 V}{\partial S_1^2} + \sigma_2^2 S_2^2 \frac{\partial^2 V}{\partial S_2^2}\right]$$

$$+ \rho\sigma_1\sigma_2 S_1 S_2 \frac{\partial^2 V}{\partial S_1 \partial S_2} - rV = 0 \tag{2}$$

Thus, equation (2) represents the standard two-dimensional BSMPDE considering the transactional costs(C) , volatility (σ) and correlation between the two assets (ρ). Using the transformations of independent variables [1] for example transforming from S_1 , S_2:
$x = In\left(S_1\right) - \left(r - \frac{1}{2}\delta_1^2\right)t$ and $y = In\left(S_2\right) - \left(r - \frac{1}{2}\delta_2^2\right)t$

Therefore, it will be transformed to:

$$\frac{\partial x}{\partial S_1} = \frac{1}{S_1}, \quad \frac{\partial x}{\partial S_2} = 0, \; \frac{\partial y}{\partial S_1} = 0,$$
$$\frac{\partial y}{\partial S_2} = \frac{1}{S_2}, \frac{\partial t}{\partial S_1} = 0, \frac{\partial t}{\partial S_2} = 0 \tag{3}$$

Transforming the PDEs using the chain rule and replacing equation (3) into it from the 2D BSMPDE to obtain;

$$\frac{\partial V}{\partial S_1} = \frac{\partial V}{\partial x}\frac{\partial x}{\partial S_1} + \frac{\partial V}{\partial y}\frac{\partial y}{\partial S_1} + \frac{\partial V}{\partial t}\frac{\partial t}{\partial S_1} = \frac{1}{S_1}\frac{\partial V}{\partial x} \tag{4}$$

$$\frac{\partial V}{\partial S_2}=\frac{\partial V}{\partial x}\frac{\partial x}{\partial S_2}+\frac{\partial V}{\partial y}\frac{\partial y}{\partial S_2}+\frac{\partial V}{\partial t}\frac{\partial t}{\partial S_2}=\frac{1}{S_2}\frac{\partial V}{\partial y} \tag{5}$$

$$\frac{\partial^2 V}{\partial S_1^2}=\frac{\partial}{\partial S_1}\left[\frac{\partial V}{\partial S_1}\right]=\frac{\partial}{\partial S_1}\left[\frac{1}{S_1}\frac{\partial V}{\partial x}\right]=\frac{1}{S_1}\frac{\partial}{\partial S_1}\left[\frac{\partial V}{\partial x}\right]=\frac{1}{S_1}\frac{\partial}{\partial x}\left[\frac{1}{S_1}\frac{\partial V}{\partial x}\right]=\frac{1}{S_1^2}\frac{\partial^2 V}{\partial x^2} \tag{6}$$

$$\frac{\partial^2 V}{\partial S_2^2}=\frac{\partial}{\partial S_2}\left[\frac{\partial V}{\partial S_2}\right]=\frac{\partial}{\partial S_2}\left[\frac{1}{S_2}\frac{\partial V}{\partial y}\right]=\frac{1}{S_2}\frac{\partial}{\partial S_2}\left[\frac{\partial V}{\partial y}\right]=\frac{1}{S_2}\frac{\partial}{\partial y}\left[\frac{1}{S_2}\frac{\partial V}{\partial y}\right]=\frac{1}{S_2^2}\frac{\partial^2 V}{\partial y^2} \tag{7}$$

$$\frac{\partial^2 V}{\partial S_1\partial S_2}=\frac{\partial}{\partial S_1}\left[\frac{\partial V}{\partial S_2}\right]=\frac{\partial}{\partial S_1}\left[\frac{1}{S_2}\frac{\partial V}{\partial y}\right]=\frac{1}{S_2}\frac{\partial}{\partial S_1}\left[\frac{\partial V}{\partial y}\right]=\frac{1}{S_2}\frac{\partial}{\partial y}\left[\frac{\partial V}{\partial S_1}\right]=\frac{1}{S_2}\frac{\partial}{\partial y}=\frac{1}{S_1S_2} \tag{8}$$

Substituting the derivatives above into equation (8) and assuming $C_1=C_2=C$ which leads to:

$$\frac{\partial V}{\partial t}+(r-C)\left[\frac{\partial V}{\partial x}+\frac{\partial V}{\partial y}\right]+\frac{1}{2}\left[\sigma_1^2\frac{\partial^2 V}{\partial x^2}+\sigma_2^2\frac{\partial^2 V}{\partial y^2}\right]+\rho\sigma_1\sigma_2\frac{\partial^2 V}{\partial x\partial y}-rV=0 \tag{9}$$

2.1 Dufort and Frankel difference scheme

The Dufort-Frankel scheme, proposed by [10] makes use of various themes discussed in Richardson and MADE schemes. It came into existence in an effort to address the instability associated with the Richardson scheme. The scheme is explicit, unconditionally stable and second order accurate in both the spatial and temporal dimensions. The Dufort-Frankel scheme is conditionally consistent with the partial differential equation it solves. It is unconditionally stable and easier to parallelize on high performance computer systems. However, since the Dufort-Frankel scheme is a two-step method, calculating the first temporal vector after the initial boundary requires some other method. The Dufort-Frankel scheme makes use of a time derivative estimation similar to the Richardson scheme.

2.2 Discretization of Black- Scholes -Merton equation

The governing equations for this particular problem are nonlinear in nature and there exists no analytical method of solving them hence a suitable numerical method is used. Finite method was used in this case.

2.3 Dufort and Frankel numerical scheme

We discretize the Black-Scholes-Merton option pricing partial differential equation (9) and form a Dufort- Frankel numerical scheme which we eventually solve using the finite difference method. Equation (9) is discretized to study the effects of transaction cost C, for call and put option values. In the Dufort-Frankel numerical scheme,V_t, V_{xx} , V_{yy} and V_{xy} are replaced by the central finite approximations but the value of $V_{i,j}^n$ in V_{xx} and V_{yy} are replaced by $\left(V_{i,j}^{n+1}+V_{i,j}^{n-1}\right)$ difference approximation. When these approximations are substituted into equation (9), and let $\sigma_1=\sigma_2=\sigma$, we get

$$\frac{V_{i,j}^{n+1}-V_{i,j}^{n-1}}{2\Delta t}+(1-C)\left[\frac{V_{i+1,j}^n-V_{i-1,j}^n}{2\Delta x}+\frac{V_{i,j+1}^n-V_{i,j-1}^n}{2\Delta y}\right]+$$
$$\frac{\sigma}{2}\left[\frac{V_{i+j,j}^n-\left(V_{i,j}^{n+1}+V_{i,j}^{n-1}\right)+V_{i-j}^n}{(\Delta x)^2}\right]+\frac{\sigma}{2}\left[\frac{V_{i,+1}^n-\left(V_{i,j}^{n-1}+V_{i,j}^{n-1}\right)+V_{i,j-1}^n}{(\Delta y)^2}\right]$$

$$+\sigma^2\left[\frac{V_{i+1,+1}^n-2V_{i-1,+1}^n-V_{i+1j+1}^n+V_{i-1,-1}^n}{4\Delta x)(\Delta y)}\right]-V_{i,j}^n=0 \tag{10}$$

Taking $\phi=\frac{\Delta t}{(\Delta x)^2}=\frac{\Delta t}{(\Delta y)^2},\Delta x=\Delta y$ on a square mesh and multiplying by 4Δ
t throughout equation (10) and re-arranging, we get the scheme;

$(2-4\sigma)V_{i,j}^{n+1} = 0.04V_{i,j}^{n} - (0.2 - 0.2C + 2\sigma)V_{i+1,j}^{n} - (0.2 - 0.2C + 2\sigma)V_{i-1,j}^{n} + (2+2\sigma)V_{i,j}^{n-1} - (0.2 - 0.2C + 2\sigma)V_{i,j+1}^{n} - (0.2 + 0.2C + 2\sigma)V_{i,j-1}^{n} - \sigma^2 V_{i+1,j+1}^{n} + 2\sigma^2 V_{i-1,j+1}^{n}$

$$+\sigma^2 V_{i+1,j-1}^{n} - \sigma^2 V_{i-1,j-1}^{n} \tag{11}$$

Taking , $\Delta x = \Delta y = 0.1$, $\sigma = 0.1$ $\rho = r = 1$ and $\Delta t = 0.01, \Rightarrow \phi = 1$ and multiply by 10 we get the Dufort-Frankel scheme

$$V_{i,j}^{n+1} = 0.25V_{i,j}^{n}\frac{(2-C)}{8}\left(V_{i+1,j}^{n} + V_{i-1,j}^{n} + V_{i,j+1}^{n} + V_{i,j-1}^{n}\right) + \frac{(1.1)}{(1-02)}V_{i,j}^{n-1} + 0.005$$

$$\left(V_{i+1,j-1}^{n} + 2V_{i-1,j+1}^{n} - V_{i+1,j+1}^{n} - V_{i-1,j-1}^{n}\right) \tag{12}$$

Taking $n = 2, i = 2.\ldots.6, j = 1$, i.e. $S1 = S2$ the above scheme in equation (12) can be written in matrix form as

We use the initial conditions below in (13)

$$\begin{bmatrix} V_{1,1}^{2} \\ V_{2,1}^{2} \\ V_{3,1}^{2} \\ V_{4,1}^{2} \\ V_{5,1}^{2} \\ V_{6,1}^{2} \end{bmatrix} = \begin{bmatrix} 0.25 & -\frac{(2-C)}{8} & 0 & 0 & 0 & 0 \\ -\frac{(2-C)}{8} & 0.25 & -\frac{(2-C)}{8} & 0 & 0 & 0 \\ 0 & -\frac{(2-C)}{8} & 0.25 & -\frac{(2-C)}{8} & 0 & 0 \\ 0 & 0 & -\frac{(2-C)}{8} & 0.25 & -\frac{(2-C)}{8} & 0 \\ 0 & 0 & 0 & -\frac{(2-C)}{8} & 0.25 & -\frac{(2-C)}{8} \\ 0 & 0 & 0 & 0 & -\frac{(2-C)}{8} & 0.25 \end{bmatrix} \begin{bmatrix} V_{1,1}^{1} \\ V_{2,1}^{1} \\ V_{3,1}^{1} \\ V_{4,1}^{1} \\ V_{5,1}^{1} \\ V_{6,1}^{1} \end{bmatrix} + \begin{bmatrix} -\frac{(2-C)}{8}\left(V_{1,2}^{1} + V_{1,0}^{1}\right) + 1.375V_{1,1}^{0} + 0.005\left(V_{2,0}^{1} + 2V_{0,2}^{1} - V_{2,2}^{1} - V_{1,0}^{1}\right) \\ -\frac{(2-C)}{8}\left(V_{2,2}^{1} + V_{2,0}^{1}\right) + 1.375V_{2,1}^{0} + -0.005\left(V_{3,0}^{1} + 2V_{1,2}^{1} - V_{3,2}^{1} - V_{2,0}^{1}\right) \\ -\frac{(2-C)}{8}\left(V_{3,2}^{1} + V_{3,0}^{1}\right) + 1.375V_{3,1}^{0} + -0.005\left(V_{4,0}^{1} + 2V_{2,2}^{1} - V_{4,2}^{1} - V_{3,0}^{1}\right) \\ -\frac{(2-C)}{8}\left(V_{4,2}^{1} + V_{4,0}^{1}\right) + 1.375V_{4,1}^{0} + -0.005\left(V_{5,0}^{1} + 2V_{3,2}^{1} - V_{5,2}^{1} - V_{4,0}^{1}\right) \\ -\frac{(2-C)}{8}\left(V_{5,2}^{1} + V_{5,0}^{1}\right) + 1.375V_{5,1}^{0} + -0.005\left(V_{6,0}^{1} + 2V_{4,2}^{1} - V_{6,2}^{1} - V_{5,0}^{1}\right) \end{bmatrix} \tag{13}$$

$$\mathrm{V(x,y,0) = 0, t = 0, x \geq y} \tag{14}$$

The boundary conditions for call asset option are

$$\mathrm{V(x,0,t) = V(x,2,t) = 0,\ V(x,1,1) = e^{x}, t > 0, x \geq y} \tag{15}$$

While the boundary conditions for put asset option are

$$\mathrm{V(x,0,t) = V(x,2,t) = 0,\ V(x,1,1) = e^{-x}, t > 0, x \geq y} \tag{16}$$

For call option we use the conditions in equation (15) so that the matrix equation (13) becomes

$$\begin{bmatrix} V_{1,1}^{2} \\ V_{2,1}^{2} \\ V_{3,1}^{2} \\ V_{4,1}^{2} \\ V_{5,1}^{2} \\ V_{6,1}^{2} \end{bmatrix} = \begin{bmatrix} 0.25 & -\frac{(2-C)}{8} & 0 & 0 & 0 & 0 \\ -\frac{(2-C)}{8} & 0.25 & -\frac{(2-C)}{8} & 0 & 0 & 0 \\ 0 & -\frac{(2-C)}{8} & 0.25 & -\frac{(2-C)}{8} & 0 & 0 \\ 0 & 0 & -\frac{(2-C)}{8} & 0.25 & -\frac{(2-C)}{8} & 0 \\ 0 & 0 & 0 & -\frac{(2-C)}{8} & 0.25 & -\frac{(2-C)}{8} \\ 0 & 0 & 0 & 0 & -\frac{(2-C)}{8} & 0.25 \end{bmatrix} \begin{bmatrix} 0.36789 \\ 0.13533 \\ 0.04978 \\ 0.018316 \\ 0.006738 \\ 0.002479 \end{bmatrix} \tag{17}$$

For put option we use the conditions in equation (16) so that the matrix equation (13) becomes

$$\begin{bmatrix} V_{1,1}^2 \\ V_{2,1}^2 \\ V_{3,1}^2 \\ V_{4,1}^2 \\ V_{5,1}^2 \\ V_{6,1}^2 \end{bmatrix} = \begin{bmatrix} 0.25 & -\frac{(2-C)}{8} & 0 & 0 & 0 & 0 \\ -\frac{(2-C)}{8} & 0.25 & -\frac{(2-C)}{8} & 0 & 0 & 0 \\ 0 & -\frac{(2-C)}{8} & 0.25 & -\frac{(2-C)}{8} & 0 & 0 \\ 0 & 0 & -\frac{(2-C)}{8} & 0.25 & -\frac{(2-C)}{8} & 0 \\ 0 & 0 & 0 & -\frac{(2-C)}{8} & 0.25 & -\frac{(2-C)}{8} \\ 0 & 0 & 0 & 0 & -\frac{(2-C)}{8} & 0.25 \end{bmatrix} \begin{bmatrix} 2.71828 \\ 7.38906 \\ 20.0855 \\ 54.59518 \\ 148.41316 \\ 403.42879 \end{bmatrix} \tag{18}$$

Solving the above matrix equation (17) and (18), we get the solutions for effects of transaction cost C for call and put option values to get results.

3 Results and Discussions

The simulation results given focus on the effects of transaction cost C, on call and put asset option values.

3.1 Effects of Transaction Cost on call option value

We solve equation (17) using MATLAB and get the results of the effects transaction cost on call option value as shown in table 1 below:

Table 1. Value of a call option fluctuation for varying Transaction Cost.

Table 1. Call option values at varying transaction cost

2* Transaction Cost	Asset Price, S ($)					
	50	**60**	**70**	**80**	**90**	**100**
C = 5	1.376309	1.4764298	1.9500810	3.1380863	5.359598	3.33276
C = 10	1.3495107	1.4272920	1.71495071	2.4380863	4.366086	2.30533
C = 15	1.337205	1.3947286	1.61160778	1.8632070	3.602757	1.758076
C = 20	1.324810	1.38676281	1.32183076	1.1471211	2.839362	1.6106954

The above results in table 1 are presented in Fig. 1.

In Fig. 1, it is observed that as transaction cost increases ($C = 5, C = 10, C = 15$ and $C = 20$) the call option value (V) decreases, for example, at S = 90, V will be; ($V = 5.359598, V = 4.366086, V = 3.602757$ and $V = 2.839362$). The call option-asset price graph and transaction cost shows an inverse relation, that is, as transaction costs increase, call option values decrease. This graph is not a perfect linear relationship due to the varied impacts of transaction costs based on market conditions such as trading frequency and other related factors. Transaction cost can be described as fees and expenses incurred when trading. Examples of such costs are bid-ask spreads, market impact costs, brokerage commission among others. Brokerage commission as a transaction cost is the fees charged by brokers for executing trades. An increase in commission in executing a trade may result into an increase in the cost of trading option consequently reducing the option value. Another factor that can bring about an increase in transaction cost is the bid-ask spread. Bid price is the highest price a buyer is willing to pay for the security while ask price is the price at which the sellers are willing to sell a particular security. The difference between the bid price and the ask price is the bid-ask spread. When there is a wider bid-ask spread, the cost of entering and exiting positions increase. This consequently reduces the profitability of option trades. The larger spreads implies that the traders have to pay a higher premium to buy options. This reduces option value. Market impact cost as an aspect of transaction cost can also reduce call option value. Market impact cost can be defined as additional cost that a trader must pay over the initial price due to market slippage.

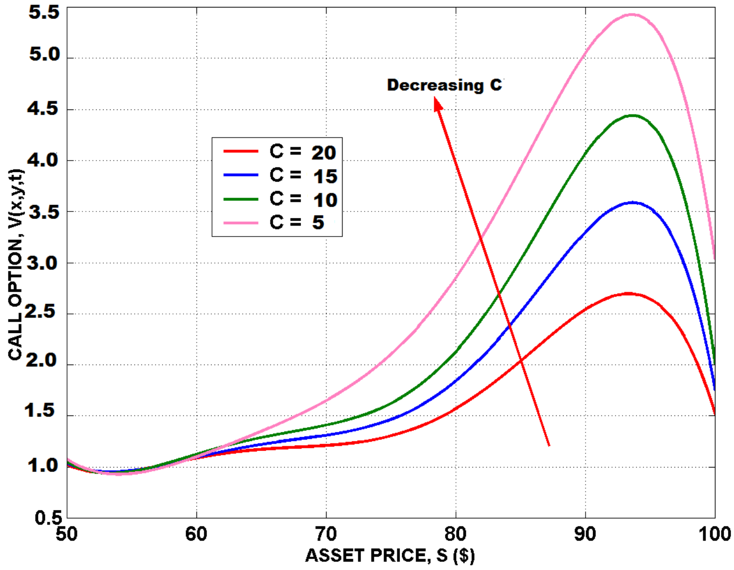

Fig. 1. Graph of call option value against asset price at varying transaction cost

It is the cost incurred because the transaction itself changed the price of the asset. Larger market impact costs adversely affect price movements and increased transaction costs. Such costs consequently reduce call option values. The increase in transaction cost may discourage traders from buying an option. This could result into reduced liquidity and lower trading volumes in the option market. Increased transaction costs also reduce the possible profit that can be made from a call option. The higher the transaction cost the lower the potential profit and call option value. Incorporation of transaction costs such as widening bid-ask spread, factoring in commission or taking into account market impacts, when valuing options can lead to significant changes in call option values. These aspects of transaction costs can affect the calculated option value and consequently trading strategies and risk management decisions. Investors and traders should be aware of transaction cost and consider their impact when evaluating option value.

3.2 Effects of Transaction Cost on put option value

We solve equation (18) using MATLAB and get the results of the effects transaction cost on call option value as shown in table 2 below

Table 2. Put option values at varying Transaction Cost

2* Transaction Cost	Asset Price, S ($)					
	50	**60**	**70**	**80**	**90**	**100**
C = 5	14.48008102	34.0498281	15.5760762	8.72862070	6.97235625	5.148657467
C = 10	12.14677105	26.0665286	12.342778	7.80036113	5.8696874	5.13705760
C = 15	9.81338107	19.032292	9.3093794	6.87202157	5.48905863	5.04536112
C = 20	7.48008109	12.0994298	7.6760810	5.64366981	5.11736954	5.01360035

The above results in table 2 are presented in Fig. 2.

In Fig. 2, it is observed that as transaction cost increases ($C = 5, C = 10, C = 15$ and C =20) the put option value (V) decreases, for example, at S=90, V will be; (V =6.97235625, V =5.8696874, V =5.48905863 and V =5.11736954). The put option value-asset price graph shows a negative gradient with an inverse relationship between put option value, asset price and transaction cost. Transaction costs such as brokerage commission charged by brokers for executing trades may affect put option value. An increase in commission in executing a trade may result into an increase in the cost of trading option. The increase in the cost of trading eventually reduces possible profit and put option value. Another aspect which leads to high transaction cost is the trading frequency. High trading frequency brings about high transaction cost. This is attributed to increased trading activities. The multiple number of transactions reduction in potential put option value with time. When transaction cost increases, the put option value decreases, this comes as a result of widening bid-ask spreads. The widened bid-ask spread reduces the probability of profit Increased transaction cost discourage traders from selling put options leading to reduced liquidity and lower trading volumes in the options market. Higher transaction costs reduce potential profit that can be made from put option. It is therefore important to consider transaction cost when evaluating the profitability and feasibility of options.

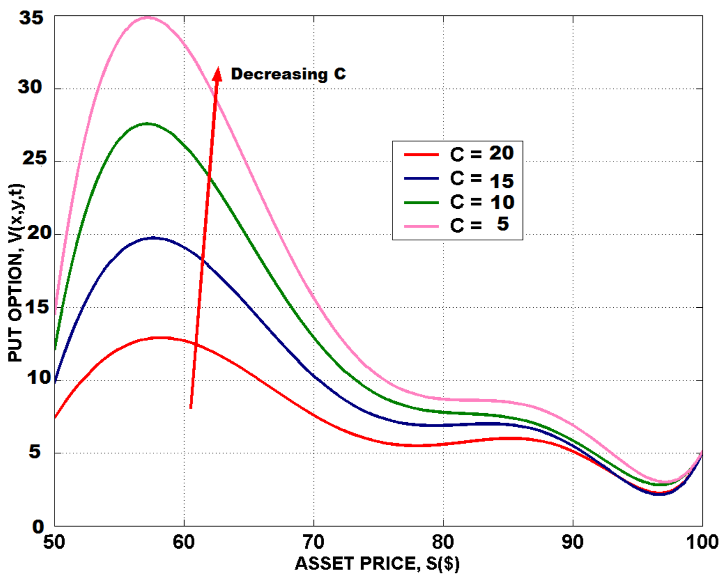

Fig. 2. Graph of put option value against asset price at varying Transaction Cost

4 Conclusion and Recommendation

4.1 Conclusion

The results from this study lead to the conclusion that incorporating the transaction cost on asset price results into a decrease on the value of both call and put option. Transaction cost can cause discrepancies between theoretical and actual option prices. It is therefore essential for traders to consider transaction costs when analyzing and trading options to ensure that they are making informed decisions. Option pricing models such as Black-Scholes model can therefore be modified to include transaction costs so as to achieve a more accurate asset pricing.

4.2 Recommendation

When pricing options, transactions costs need to be incorporated. This ensures a more accurate representation of real world trading scenarios. The findings from this study can be used by the potential investors to accurately compute possible returns on investment. To the government, the research findings can be used in formulating policies governing price control in stock exchange market.

5 Suggestion for Further Studies

The research recommends future work should consider a case of three-dimensional BSMPDE with transaction cost for American option.

Competing Interests

The authors declare that no competing interest regarding the publication of this paper.

References

[1] Ndede CO, Sigey JK, Abonyo JO, Menge B, Marigi EM. Investigation of effects of asset price fluctuations on option value. Global Journal of Pure and Applied Mathematics. 2019;15(6);909-923.

[2] Morales-Bañuelos P, Muriel N, Fernández-Anaya G. A modified Black-Scholes-Merton model for option pricing. Mathematics. 2022;10(9);1492.

[3] Company R, Navarro E, Pintos J, Ponsoda E. Numerical solution of linear and nonlinear Black-Scholes option pricing equations. Computers Mathematics With Applications. 2008;56(3):813-821.

[4] Kumar S, Yildirim A, Khan Y, Jafari H, Sayevand K, Wei L. Analytical solution of fractional Black-Scholes European option pricing equation by using Laplace transform. Journal of Fractional Calculus and Applications. 2012;2(8):1-9.

[5] Tseng SH, Nguyen TS, Wang RC. The Lie algebraic approach for determining pricing for trade account options. Mathematics. 2021;9(3):279.

[6] Onyango SN, Omollo Ongati N, Otula NJ. Walrasian-Samuelson price adjustment model. International Journal of Mathematics & Computation. 2010;9(D10):46-58.

[7] Aguilar J, Korbel J. Simple formulas for pricing and hedging European options in the finite moment log-stable model. Risks. 2019;72(2):36.

[8] Chowdhury R, Mahdy M, Alam T, Al Quaderi G, Rahman M. Predicting the stock price of frontier markets using machine learning and modified Black-Scholes Option pricing model. Physica A: Statistical Mechanics and Its Applications. 2020;555:124444.

[9] Hossan M, Islam M, Kamrujjaman M. Efficient numerical schemes for computations of European options with transaction costs. European Journal of Mathematical Analysis. 2022;2;2-15.

[10] Gasparin S, Berger J, Dutykh D, Mendes N. Stable explicit schemes for simulation of nonlinear moisture transfer in porous materials. Journal Of Building Performance Simulation. 2018;11:129-144.

Peer-review history:
The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)
https://www.sdiarticle5.com/review-history/113446