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On the Convective Mechanism of Fluctuations in the Luminosity of Semi-Controlled Variable Stars

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Abstract

The mechanisms of changing the luminosity of various types of stars are considered. The problem of supercritical convection in an infinite plane layer is solved in the approximation of low velocities, the plane equation of which coincides with the equation for high velocities, a stationary solution is obtained. It is shown that there minimum two mechanisms of luminosity oscillations in stars: Supercritical convection in the upper layers of the star generates oscillatory changes in temperature and, accordingly, in the luminosity of the star, on the other hand, convection may have asymmetric periods. In the post-Newtonian approximation, the estimate of the frequency of changes in the luminosity of stars is corrected, and a general formula is obtained.

Subject Areas

Astrophysics, Cosmology

Keywords

Helium, Convection, Temperature, Gravity, Frequency

1. Introduction

The luminosity of stars in various aspects, in terms of the emission lines of chemical elements [1], the intensity of the total luminosity or luminosity fluctuations is intensively studied.

The values of the main parameters of stars, such as luminosity and central temperature, can be obtained from the basic equations of the theory of stability and equilibrium in stars. However, some results are not satisfactory, it is necessary to obtain a new analytical scheme. From the theory of the internal structure and stability of stars, it is possible to obtain a dependence for the central temperature of any gaseous star [2]. There remains the question of fluctuations in the luminosity of stars.

In the absence of gravity in the static case, the temperature in a spherical system with a heat source in the center, as follows from Fourier's law, varies by hyperbola. It is argued that in pulsating variable stars, a change in volume can occur due to the instability between pressure and the force of gravity. This is doubtful because the pressure increases during compression, for example, according to the adiabatic law $pV^k = const$, где $k = C_p/C_V$. That is, the pressure is inversely proportional to r^{3k}, whereas the force of gravity decreases inversely proportional to r^2. Thus, an equilibrium radius arises, and the equilibrium between pressure and gravity is stable. For RR Lyra and cepheid type variables, the Kappa luminosity change mechanism associated with He_3 ionization is complemented by a gamma mechanism associated with a lower temperature of ionized helium and a radius mechanism associated with a decrease in the surface area of the star during compression and an increase in its density.

For semiregular variables, the change in luminosity is associated with convection in the outer layers, but it has not been investigated how the periodicity of luminosity arises.

Convection in stars is usually considered within the framework of classical thermodynamics. It is believed that in the equilibrium state, heat transfer occurs only through radiation, photons are absorbed by atoms and re-emitted, or radiation is scattered by free electrons. Convective mixing is determined only by temperature gradients. This approach leads to a contradictory result. Near the surface of the Sun, the thickness of the convective layer can reach about 10^4 km, in K-class stars it is noticeably larger, in M-stars it can reach almost to the center, in stars hotter than F_0, the convective layer is practically absent. It turns out that the hotter the star, the less convection.

To study convection in stars without affecting the nuclear convective zone, it is necessary to consider a complete system of convection equations, for example, in the Boussinesq-Oberbeck approximation. equations of thermogravitational convection, equations of magnetic hydrodynamics [3], plasma equations [4] [5], So the theoretical investigation of it is too difficult task.

Convective flows occur if the Rayleigh number is greater than a certain critical value, the equilibrium of the liquid becomes unstable. The symmetry of the infinite horizontal layer and homogeneous heating conditions make possible periodic structures—shafts, rectangular and hexagonal cells, which are unstable with respect to three-dimensional, two-dimensional long-wave longitudinal disturbances, etc. In the case of a limited horizontal layer, profiles of the soliton type are possible. In the supercritical region, linear theory allows you to choose a perturbation whose growth rate is maximal, stationary solutions of the nonlinear problem are found in a wide range of Rayleigh number variations, allowing you to determine the intensity of motion, heat flow, etc. It was found that the additional heat flow in the horizontal layer near the threshold increases linearly with increasing supercriticality.

The mechanisms of convection pulsations of semi-correct variables are the least studied [6].

In [7], the Stothers-Leung theory was further developed, according to which the pulsations of the luminosity of stars are associated with the sequentially synchronous inversion of giant convection cells in the shells of massive red supergiants. The data in [7] are in good agreement with the data on long secondary periods of fluctuations in the luminosity of two stars, Betelgeuse and Antares.

In this article, alternative mechanisms of fluctuations in the pulsations of the luminosity of stars are proposed.

Some points of it can be shown using a simple model of convection equations in the Boussinesq approximation.

2. Model Task

Consider three-dimensional convection in a horizontal layer with heat-insulating surrounding arrays at small exceedances of the critical Rayleigh number. For simplicity and brevity, we investigate a linearized system in which the speeds are low. In the heat equation, we take into account the nonlinear gradient term. Then the system of convection equations will take the form

$$\begin{cases} \nabla p - U + R_p T g = 0 \\ -\dfrac{\partial T}{\partial t} + \Delta T + Ug - U\nabla T = 0 \\ divU = 0 \end{cases}$$

where $R_p = \dfrac{g\beta L^2 \delta T}{\nu\chi}$, T-temperature deviation from the equilibrium distribution, δT -temperature difference at the boundaries of the layer, χ-thermal conductivity, ν-kinematic viscosity; unit of length L is the width of the layer, velocity is measured in *x/L*, pressure is measured in $\rho_0 \nu\chi$.

Let's choose a suitable coordinate system, then for the thermally insulated layer the boundary conditions will be written as

$$z = 0,\ z = 1;\ U_z = \partial_z T = 0$$

Excluding the velocity from the equations, we get the system:

$$\begin{cases} -\dfrac{\partial T}{\partial t} + \Delta T + \nabla p \nabla T - RT\dfrac{\partial T}{\partial z} + \dfrac{\partial P}{\partial z} + RT = 0 \\ -\Delta p + R\dfrac{\partial T}{\partial z} = 0 \end{cases} \tag{1}$$

Since we are solving a model problem, for this system we can choose boundary conditions in this form:

$$z = 0;\ z = 1;\ \frac{\partial T}{\partial z} = 0;\ \nabla P = RT\vec{g} \quad \text{или} \quad \frac{\partial P}{\partial z} = -RT\ .$$

For this system, the solvability condition of an inhomogeneous system of differential equations is obtained by integrating the expression for $\partial_z^2 T_k$, which is obtained in k-th order, by the thickness of the layer.

To consider processes having different time and spatial scales, we use the method of many scales. The functions of pressure and temperature can be represented as depending on a set of variables $t_n = a_m^n t$, where t_1, t_2, etc. are slow times; then the time derivative has the form:

$$\frac{\partial}{\partial t} = \sum_{n=0}^{\infty} a_m^n t_n$$

In the future, we believe that all the fast processes have already passed and the functions T, P do not depend, at least, on t_0.

For spatial variables (horizontal), we restrict ourselves to the first order in the series, assuming that in the zero-order the functions T, P have no dependence on x_0, y_0:

$$\xi = a_m x, \quad \eta = a_m y$$

Then the derivatives of these coordinates have the form:

$$\frac{\partial}{\partial x} = a_m \frac{\partial}{\partial \xi}; \quad \frac{\partial}{\partial y} = a_m \frac{\partial}{\partial \eta}$$

We use the small parameter method; with small supercriticities $R - R_0$ the wave number, temperature, pressure functions, as well as the critical Rayleigh number are decomposed in a series by degrees of the wave number:

$$R_p = \sum_{n=0}^{\infty} a_m^{2n} R_{2n}; \quad T = \sum_{n=0}^{\infty} a_m^4 T_n; \quad P = \sum_{n=0}^{\infty} a_m^4 P_n \tag{2}$$

Let's substitute these derivatives and decomposition (2) into the system of equations and into the system of Equation (1). Using boundary conditions, we obtain in zero order:

$$P_0 = R_0 T_0 \cdot z - \frac{R_0 T_0}{2}; \quad \frac{\partial T_0}{\partial z} = 0$$

In the first order, we obtain that the functions T, P do not depend on t_1;

$$P_1 = R_0 T_1 \cdot z - \frac{R_0 T_1}{2}; \quad \frac{\partial T_1}{\partial z} = 0$$

In the second order, using the solvability condition, we obtain: $R_0 = 12$, the functions do not depend on t_2;

$$P_2 = \frac{R_0}{2} \Delta_2 T_0 \left(\frac{z^2}{2} - \frac{2}{3} z^3 + \frac{z^4}{2} - \frac{z^5}{5} \right) + \frac{R_0}{2} \left[\nabla_2 T_0 \right]^2 \left(2z^3 - z^4 \right) + R_2 T_0 z + C_2$$

where Δ_2, ∇_2 —two-dimensional Laplacian and gradient, C_2 —integration constant independent of z,

$$T_2 = \Delta_2 T_0 \left(-\frac{z^2}{2} + z^3 - \frac{z^4}{2} \right) + \left(\nabla_2 T_0 \right]^2 \left(3z^2 - 2z^3 \right)$$

In the third order, the calculations coincide with the calculations in the second order. We get:

$$P_3 = \frac{R_0}{2}\Delta_2 T_1\left(\frac{z^2}{2} - \frac{2}{3}z^3 + \frac{z^4}{2} - \frac{z^5}{5}\right) + \frac{R_0}{2}\left[\nabla_2 T_1\right]^2\left(2z^3 - z^4\right) + R_2 T_1 z + C_3$$

$$T_3 = \Delta_2 T_1\left(-\frac{z^2}{2} + z^3 - \frac{z^4}{2}\right) + \left[\nabla_2 T_1\right]^2\left(3z^2 - 2z^3\right)$$

the functions T, P do not depend on t_3.

In the fourth order, using the solvability condition, we obtain a closed equation for T_0:

$$\frac{\partial T_0}{\partial t} + \frac{2}{21}\Delta_2^2 T_0 + \frac{R^2}{12}\Delta_2 T_0 - \frac{6}{5}\left(\nabla_2 T_0 \nabla_2\right)\left[\nabla_2 T_0\right]^2 - \frac{6}{5}\Delta_2 T_0\left[\nabla_2 T\right]^2 = 0 \quad (3)$$

Since the solution of a linear problem should have the form

$$T_0 = \exp(i\xi)\exp(i\eta)$$

so, substituting this expression in (3), we get $R_2 = \frac{8}{7}$.

To get rid of the coefficients in Equation (3), we introduce variables

$$\tau = \frac{2}{\sqrt{35}}t, \quad \bar{T} = \sqrt{\frac{63}{5}} \cdot T_0$$

then Equation (3) will be written as::

$$\frac{\partial \bar{T}}{\partial \tau} + \Delta_2^2 \bar{T} + \Delta_2 \bar{T} - \left(\nabla_2 \bar{T} \nabla_2\right)\left[\nabla_2 \bar{T}\right]^2 - \Delta_2 \bar{T}\left[\nabla_2 \bar{T}\right]^2 = 0$$

In a planar problem, when only one horizontal coordinate is involved, the equation goes into the following:

$$\dot{\vartheta} + \partial_\xi^4 \vartheta + \partial_\xi^2 \vartheta - \partial_\xi^2\left(\vartheta^3\right) = 0 \quad (4)$$

where $\vartheta = \partial_\xi \bar{T}$. For high speeds, the calculations are much more cumbersome, but the equation turns out exactly the same. In the stationary case, equation (4) has the solution

$$\vartheta = \sqrt{\frac{2k^2}{1+k^2}}\,\mathrm{sn}\frac{\xi - \xi_0}{\sqrt{1+k^2}}$$

where k is the module of the elliptic Jacobi function associated with the wavenumber $\bar{a} = a/a_m$ by the relation $\bar{a} = \frac{\pi}{2K(k)\sqrt{1+k^2}}$, where $K(k)$ is a complete elliptic integral of the first kind, ξ_0 is an integration constant.

Numerical analysis (4) shows that the temperature increases with time to the upper boundary of the layer.

That is: equation (4) itself does not contain vibration modes, but it shows a periodic cessation of convection.

Are large exceedances of the Rayleigh number of critical value characteristic for stars? *i.e.* the analytical solution of the Rayleigh-Benard problem for a homogeneous one-component liquid was investigated in [8], distributions of temperature and velocity fields for supercriticality up to $R = 3\times10^6$ are obtained.

For semi-controlled supergiants, the acceleration of gravity, which is included in the Rayleigh number, exceeds the terrestrial by at several orders of magnitude, so this calculation is valid for them with small supercritical exceedances.

3. Dependence of the Oscillation Period on the Rayleigh Number

It can be seen from the equations of the Boussinesq model that the harmonic dependence of temperature on time is realized in a linear system.

Oscillatory modes in various convective nonlinear systems are well known.

Vibrational regimes were experimentally observed during Marangoni concentration convection [9]. Forced oscillations were studied in a system with magnetic convection, see [10]. The oscillatory mode of Rayleigh-Benard convection is investigated, and the critical value of the Rayleigh number is obtained in a linear system of nonautonomous Lorentz-type equations.

Asymmetric oscillations in a convective system are considered in [11], and the dependence of the natural frequency of the system on the normalized Rayleigh number is obtained in the Lorentz model for Pr = 10.

$\omega \approx 0.22 + 2.11\sqrt{r-1.4}$; $1.5 < r < 20$. Without restrictions on generality, one can put

$$\omega = c_1 + c_4\sqrt{Ra - c_5} ,$$

where c_i—characteristic parameters.

Estimation of the frequency of changes in luminosity

$$\omega = 2\pi\sqrt{G\rho}$$

see [[12], p. 403].

The critical Rayleigh numbers depend on the wave numbers and on the dimensionless parameters of the medium: The magnetic Prandtl number, the Hartmann number, the Taylor number, the Rossby number. In massive bodies, a relativistic effect occurs.

Rayleigh number $Ra \sim g$, that is, the Rayleigh number depends on the radius of the star, due to the dependence of gravity on the distance to the center, which is enhanced due to its non-classicity. For example, the equations of motion of a material point in a gravitational field in the case of velocities small compared to the speed of light and weak constant gravitational fields, the potential and equations of motion take the form

$$\varphi = -c^2\left(1+g_{44}\right)/2, \quad \varphi(r) = -\frac{Gm}{r} + C$$

$$\Delta\varphi = \frac{2}{r}\frac{\partial\varphi}{\partial r} + \frac{\partial^2\varphi}{\partial^2 r} = 4\pi Gm;\ a \sim -\nabla\varphi .$$

In the post-Newtonian approximation $a = -\nabla\varphi + q\left(\nabla\varphi\right)^2$,

$\left(\nabla\varphi\right)^2 = \frac{1}{2}\Delta\varphi^2 - \varphi\Delta\varphi$, $a = -\nabla\varphi + 4\pi qm\varphi = g = Gm/r^2 + 4\pi qGm/r$. So

$g = \frac{4}{3}\pi G\rho r(1+r)$. Then the Rayleigh number $Ra = \frac{4\pi r(1+r)\beta L^3}{\nu\chi} G\rho$, and the natural frequency will be written as

$$\omega = c_1 + c_2\sqrt{G\rho - c_3}$$

Thus, the natural frequency of asymmetric oscillations in the convection equations corresponds to the estimation of the period of changes in the luminosity of stars.

4. Conclusions

It is generally believed that in stars, the temperature directly below the photosphere increases extremely rapidly into the interior and radiation emission cannot ensure the release of radiation from deeper hot layers, which is the cause of convection, which ensures the stability of the star.

However, when convection occurs, mixing occurs, which leads to temperature equalization.

The mechanism generating the oscillatory mode can be schematically represented as follows: 1) first, due to the outflow of energy in the form of radiation and stellar wind in the surface layer of the star, the temperature difference generates intense convective mixing, which leads to an averaging of the temperature over the layer, the temperature gradient decreases; 2) the same happens with the next, lower-lying layer; 3) the cooling of the upper layers continues to the level where thermonuclear fusion takes place, the energy of which, with the help of convection, heats the upper layers to the previous temperature, the temperature gradient decreases again. 4) The cooling of the upper layers begins again, etc.

Thus, there are at least two mechanisms of periodic pulsations of luminosity of semi-controlled variables:

- temperature equalization along the upper layer, a decrease in the Rayleigh number less than the critical one and the cessation of convection, then the occurrence of convection due to heating from the stellar core and cooling of the upper boundary of the layer;
- the occurrence of asymmetric oscillations in the convective layer.

In the latter case, taking into account the dependence of the critical Rayleigh number on the gravitational field, it is possible to adjust the formula for the pulsation frequency of semiregular variable stars.

Conflicts of Interest

The author declares no conflicts of interest.

References

[1] Hosoya, K., Itoh, Y., Oasa, Y., Gupta, R. and Sen, A. (2019) Spectroscopic Survey of Hα Emission Line Stars Associated with Bright Rimmed Clouds. *International Journal of Astronomy and Astrophysics*, **9**, 154-171.

https://doi.org/10.4236/ijaa.2019.92012

[2] Palacios, A. (2016) The Central Temperature of the Stars. *Journal of High Energy Physics, Gravitation and Cosmology*, **2**, 183-185. https://doi.org/10.4236/jhepgc.2016.22017

[3] Reiners, A. and Basri, G. (2009) On the Magnetic Topology of Partially and Fully Convective Stars. *Astronomy and Astrophysics Journal*, **496**, 787-790. https://doi.org/10.1051/0004-6361:200811450

[4] Pegoraro, F., Califano, F., Manfredi, G. and Morrison, P.J. (2015) Theory and Applications of the Vlasov Equation. *The European Physical Journal D*, **69**, Article No. 68. https://doi.org/10.1140/epjd/e2015-60082-y

[5] Kopp, M.I. (2015) On the Theory of Stability of a Rotating Plasma with a Constant Temperature Gradient. *M. Young Scientist*, **23**, 34-40. https://moluch.ru/archive/103/24082/

[6] Karttunen, H., Kroger, P., Oja, H., Poutanen, M. and Donner, K.J. (2016) Fundamental Astronomy. 6th Edition, Springer.

[7] Stothers, R.B. (2010) Giant Convection Cell Turnover as an Explanation of the Long Secondary Periods in Semiregular Red Variable Stars. *The Astrophysical Journal*, 725, 1170-1174. https://doi.org/10.1088/0004-637x/725/1/1170

[8] Gabitova, A. (1983) Numerical Study of Convective Flows in Elastic-Viscous Liquids. Ph.D. Thesis, Kazan State University.

[9] Denisova, M.O., Zuev, A.L. and Kostarev, K.G. (2022) Oscillatory Modes of Concentration Convection. *Uspekhi Fizicheskih Nauk*, **192**, 817-840.

[10] Kopp, M.I., Tur, A.V. and Yanovsky, V.V. (2020) Magnetic Convection in an Inhomogeneously Rotating Electrically Conductive Medium under the Influence of Modulation of an External Magnetic Field. *Journal of Experimental and Theoretical Physics*, **157**, 901-927.

[11] Melentyev, A.B. and Tarunin, Y.L. (2012) Effects of Asymmetric Modulations in Convection. *Computational Continuum Mechanics*, **5**, 284-291. https://doi.org/10.7242/1999-6691/2012.5.3.33

[12] Kononovich, E.V. and Moroz, I.V. (2004) General Astrophysics Course, Moscow. URSS. (In Russian)